A Framework for Connecting Natural Language and Symbol Sense in Mathematical Word Problems for English Language Learners

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Abstract

Working fluently within the multiple semiotic systems of the language of mathematics requires developing strong symbol sense and connecting meaning of symbols to meanings in natural language. Challenges can exist for English language learners (ELLs) when connecting natural language and symbolic representations, particularly in the context of a mathematical word problem. This article presents a framework that connects mathematical word problem solving stages to multiple semiotic systems while providing elements of symbol sense that ELLs can develop in order to work with mathematical word problems.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What challenges does mathematics language present to English Language Learners?
2. How is language used differently in mathematics than in other content areas?
3. What resources do English Language Learners bring to the classroom that can aid in their learning of mathematics?

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Introduction

Mathematics can be considered a language in itself, composed of natural language and a symbolic system of mathematical signs, graphs, and diagrams (Drouhard & Teppo, 2004). The learning of mathematics is heavily dependent on both the symbolic language of the discipline (including syntax and organization of symbols) and the natural language of instruction (including discourse practices specific to this discipline) (Crowhurst, 1994; Moschkovich, 2007). Halliday (1978) describes languages as *semiotic systems*, a systemic resource for meaning making. In a semiotic system, we understand what is being expressed based on prior experiences with that system. Working fluently within or between multiple semiotic systems such as natural and symbolic languages requires developing strong *symbol sense*, which includes having an awareness that one can successfully create symbolic relationships which represent written information; experiencing different roles played by symbols; and appreciating the power of symbols to display and explain relationships expressed in natural language (Arcavi, 1994, 2005).

Research and personal experiences tell us that the complexity of working in multiple semiotic systems in mathematics presents challenges for all learners. There are, however, additional linguistic demands for English Language Learners (ELLs) that make developing symbol sense and transitioning between the symbolic and natural language even more of a challenge, as they learn to filter their existing and developing knowledge of mathematical language through a second natural language (Brown, 2005). Our goal in this paper is to focus on these additional challenges for ELLs by analyzing the potential linguistic difficulties that may exist when connecting natural language and symbolic representations in mathematics, particularly in the context of a mathematical word problem.

Challenges in Mathematics for ELLs: The Case of Word Problems

An examination of the gaps between ELL and non-ELL performance in mathematics shows small gaps for strictly computational problems, but large gaps on word problems and problems that contain linguistically complex terms (Abedi, 2004). An interplay between symbolic and natural language is clearly present when solving mathematical word problems where students must be able to decode not only the language of the question and the overlaying context, but must also have knowledge of and be able to represent words with the mathematical symbols needed to effectively answer the question. It is clear that many students (both ELLs and non-ELLs) encounter difficulties with this connection between words and mathematical symbols in word problems (see e.g., Reed, 1999). Some studies, however, have discussed the additional complexity that exists for ELLs when working with many types of word problems (Celedón-Pattichis; 2003; Martiniello, 2006, 2008). While trying to work within a second language of English, ELLs must negotiate the ways in which a "third language" of mathematics symbolically represents a given problem (Brown, 2005).

Some suggested reasons for added difficulties for ELLs on word problems include: a lack of built-in contextual clues found in literary narratives (Carey, Fennema, Carpenter, & Franks, 1995), unfamiliar cultural contexts and interpretations (Solano-Flores & Trumbull, 2003), reading comprehension issues (Schleppegrell, 2007), and the artificial contexts of word problems (Wiest, 2001). See also Celedón-Pattichis (2003) for other challenges mathematics word problems present to ELLs. Many suggestions have been offered for helping ELLs work with word problems, including helping students recognize and understand keywords (e.g., more than, take away, of, per, total, etc.), modifying the language complexity of the problem, and using manipulatives (Aguirre & Bunch, 2012). Another suggestion that has been found effective for helping ELLs in mathematics is to make use of the ideas and skills that they bring with them to the classroom. This can include assessing prior knowledge to determine an ELLs' familiarity with a context, planning for the use of multiple tools and models by both the student and the teacher (e.g., visuals, diagrams, gestures) (Ramirez & Celedón-Pattichis, 2012), and using the language and cultural tools that an ELL brings to the classroom as resources for learning (Celedón-Pattichis & Ramirez, 2012).

These useful suggestions for helping ELLs may be easier to implement in some problems than in others. Consider, for example, the first part of a constructed response word problem selected from 6th grade sample items provided by the Indiana Statewide Testing for Educational Progress (ISTEP+) Grades 6 through 8 (Indiana Department of Education, 2012):

> Sue bought 4 rings for her mom. Each ring cost the same amount of money. The total cost was \$31.What is the cost per ring?

This problem contains common key words that might allow an ELL to recognize that the cost per ring is the total cost divided by the number of rings. The language is relatively simple, and knowing what "rings" are is not key to the solution of the problem. We need only recognize that we have 4 things that each cost the same and we spent \$31 in total.

Now consider another problem from the same set of sample items (Figure 1) (Indiana Department of Education, 2012). An examination of this problem shows that although some common keywords like "more" and "left" are found in the text, they are not as easily transferrable to mathematical symbols as they might be in the first example. We suggest that a problem like this requires a deeper understanding of how the larger sentence structure connects to mathematical symbols. We will revisit this problem in more detail below to look carefully at potential ways in which ELLs may struggle or succeed in working with it. We recommend that readers take a moment before continuing and solve this problem on their own, thinking carefully about the understanding of both the natural and symbolic languages needed to solve it.

Grade 7 Constructed Response Item (Alg. & Functions/Problem Solving)

Irene spent half of her weekly allowance playing miniature golf. To earn more money, her parents let her wash the car for $4.
Write an equation that can be used to determine Irene's weekly allowance (a) if she has $12 left after washing the car.

Equation______________________________

This week Irene used her allowance to buy each of her 5 friends a bracelet and still had $3 remaining.
Each bracelet cost the same amount of money.
What was the cost of 1 bracelet?
Show All Work

Figure 1. Example: ISTEP+ Mathematics Sample Items Grade 7

A Framework for Looking at Word Problems

We know that when working with mathematical word problems, ELLs need to access the language of mathematics through multiple semiotic systems that fulfill different functions: (a) natural language introduces, contextualizes, and describes a mathematical problem; (b) symbolism is used for finding the solution of the problem; and (c) visual images deal with visualizing the problem graphically or diagrammatically (de Oliveira & Cheng, 2011; O'Halloran 2004, 2005). All of these systems may involve vocabulary, sentence structures, contexts, and representations that are new or unfamiliar to ELLs (Martiniello, 2008); we have chosen to focus primarily on the first two in this paper to examine difficulties with connecting natural language and symbolic representations in word problems. We also know that, in most mathematical problem solving situations, we can break the solving procedure down into different *stages* which may include: formulating the problem from a real-world application, solving a mathematical representation (symbolic, graphical, etc.), and interpreting and checking the solution in the context of the real-world situation. When working on a mathematical problem, learners call upon the semiotic systems in different ways at different stages of problem solving.

Multiple Semiotic Systems: Natural Language and Symbolism

Natural language use in mathematics is characterized by the dominance of relational processes presented through verbs that show relationships, such as *be, have,* and *represent*, and the frequent use of nominalizations, the expression as a noun or nominal group of what would in everyday language be a verb, adjective, or conjunction (e.g., *multiplication*, *exponent*). For example, in the following test item, “What is the area, in square feet, of a circle with a diameter of 8 feet? Use 3.14 for pi,” the relational process *is* is used in the question along with the nominalization *the area of a circle with a diameter of 8 feet.* This test item is part of the ISTEP+ item sampler for grades 6-8 (Indiana Department of Education, 2012).

Mathematical content is presented using natural language to carry forward the argument (O’Halloran, 2000). Making sense of the natural language in a word problem is something with which ELLs have commonly been seen to struggle (de Oliveira & Cheng, 2011; Martiniello, 2006, 2008).

Symbolism is used in mathematics for the solution process (O’Halloran, 2000). This semiotic system is often a cause of great confusion for all students due, in part, to the multiple ways in which symbols are used. For example, symbols name, label, signify, communicate, simplify, represent, reveal structure, and display relationships (Arcavi, 1994; Kinzel, 1999; Pimm, 1995; Stacey & MacGregor, 1999). For example, when stating the often used Pythagorean Theorem, instead of continually making the cumbersome statement “the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two adjacent sides,” we label the sides of the triangle as *a, b*, and *c* (see Figure 2) and simply state $c^2 = a^2+b^2$. Here symbols make it much easier to quickly communicate and display the geometric relationship. Symbols also play multiple roles within a single mathematical statement, acting as generalized numbers, arguments of a function, parameters, unknown numbers, and variables (Usiskin, 1988). For example, in the symbolic representation for an equation of a circle, $x^2+y^2 = r^2$, r represents the radius of the circle and is a constant or parameter for the equation, while x and y are variables. In the equation $50 = 5x$, we can think of x as an unknown number rather than a variable because it can only have one value here. These numerous roles play by symbols make matters even more complicated as ELLs try to make connections between the language used to describe a mathematical problem and the symbols required to solve the problem. These symbols may exist across many different languages and understanding them is challenging regardless of one's native language, but ELLs need to draw on knowledge of a language they are still developing in order to use symbols when transitioning from words to symbols.

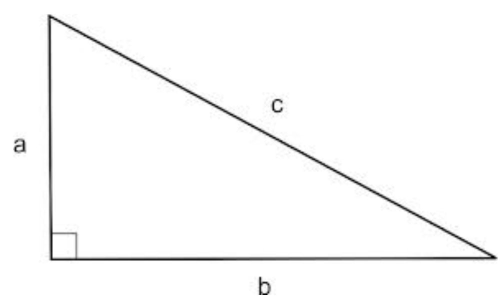

Figure 2. Using Symbols As Labels on a Triangle For The Pythagorean Theorem

When working with mathematical word problems, the interaction with natural language occurs typically in the formulation and interpreting/checking stages of problem solving while the solving stage is heavily dependent on an interaction with and manipulation of symbols (including numbers and/or letters) or analysis of visual representations. Thus, the symbolic language and visual image systems (and the links between them) play a primary role in this stage.

A Focus on Symbol Sense

Within the natural language and symbolic semiotic systems, we can identify one common important element that we choose to focus on in this paper: *symbol sense*, which Arcavi (1994) describes as "a quick or accurate appreciation, understanding, or instinct regarding symbols" (p. 31) that is involved at all stages of mathematical problem solving. Kenney (2008) has used a symbol sense framework (constructed using adaptations of work by Pierce and Stacey (2001, 2002)[1] and Arcavi (1994, 2005)), to investigate students' reasoning with mathematical symbols at different problem solving stages. In this paper, we have modified this framework to connect the problem solving stages to the semiotic systems and highlight the elements of symbol sense that ELLs may need to work with mathematical word problems (Table 1).

Table 1
A Framework for Symbol Sense for English Language Learners (ELLs)

Problem Solving Stage	**Semiotic Systems**	**Examples of Symbol Sense Required**
Formulation	Making sense of the natural language; Linking the natural language and symbolic systems	• Knowing how and when to use symbols • Knowing that symbols play different roles in different contexts • Ability to select possible symbolic representations • Knowing that chosen representation can be abandoned when they are not working
Solving	Working within the symbolism system	• Recognizing conventions and basic properties o Knowing meaning of symbols o Knowing order of operations o Knowing properties of operations
	Linking the symbolism and visual image systems	• Knowing when to abandon symbols for other visual approaches • Knowing meaning of symbols in a visual representation (e.g. labels) • Linking key features
Interpretation and Checking	Linking back to the natural language system; Making meaningful sense of how the result connects to the original question	• Linking symbol meanings to personal expectations • Linking symbol meanings to the problem • Using symbols to communicate results

Application of Framework: Examining Challenges of a Mathematics Test Item for ELLs

In this section, we use a standardized test item from the ISTEP+ Grades 6 through 8

[1]Although Pierce and Stacey's work related directly to symbol sense when working with computer algebra system (CAS) calculators, they made claims that the components should be the same when working without technology. Kenney (2008) has used the framework to explore the latter situation.

(Indiana Department of Education, 2012) to identify the potential challenges for ELLs. In the current era of teacher accountability, we know from experience that teachers are drawing heavily on sample items and practice tests from the end-of-year exams that their students will take to help prepare students for these tests. The problem we have chosen to discuss here (see Figure 1) was purposefully selected from a sample test bank because it represents a similar linguistic structure as textbook word problems that de Oliveira & Cheng (2011), as part of a larger study on the linguistic challenges of mathematics, found to be particularly difficult for ELLs in the classroom. In this paper, we apply our framework to show how the different semiotic systems connect within a mathematical problem of this type and how the resources of natural language and symbols are employed in its construction. The linguistic complexity of these types of word problems, as explained in de Oliveira (2012), makes them more likely to pose challenges for ELLs. We, therefore, explain some challenges that ELLs face in particular.

Analysis of Part 1 of the Task

An analysis of the word problem in Figure 1 shows that natural language and symbols are interconnected in students' possible solutions. This test item has two sets of tasks that students are to complete, one starting with *Irene spent half…* (we will refer to this as Part 1) and the other starting with *This week Irene used….* (Part 2). In Table 2, we break down Part 1 and connect the framework for symbol sense to a linguistic analysis of the different clauses in the task.

Table 2
Application of Framework and Linguistic Analysis of Part 1 of the I-STEP Problem

Clause in Problem	**Problem Solving Stage**	**Symbols Involved**	**Linguistic Analysis: What is required linguistically to work with the problem?**
Irene spent half of her weekly allowance playing miniature golf	Formulation	1/2	• Linking the natural language expressed in the word *half* to the numeric representation ½ • Making sense of the concept of *weekly allowance* being an amount of money
To earn more money, her parents let her	Formulation	+4	• Making sense of the concept of earning as meaning to receive more

wash the car for $4.			• Linking receiving more to addition operation • Making meaning by inferring that she will only wash the car once
Write an equation to determine Irene's weekly allowance (a) if she has $12 left after washing the car.	Formulation Interpretation	$a - ½ a + 4 = 12$ or $½ a + 4 = 12$ $a = 2(12-4)$	• Making sense of keywords in the phrase *$12 left* as indicating that this amount is what remains from allowance after washing the car • Recognizing that she earned the extra 4 dollars *after* she had spent half (it was not ½(*a*+4)). • Interpreting the produced equation with the natural language to check the meaning of what they produced

Part 1 begins by introducing a context for the situation. The concept of weekly allowance is introduced in the first clause, which may cause difficulties for ELLs who may not be familiar with this concept and may not recognize it as an amount of money. In the same clause, we also see the word *half* which students have to connect to the symbolic representation ½. The second sentence begins with a clause that indicates purpose, *To earn more money,* so ELLs have to make the connection between earning more money and the following clause, *her parents let her wash the car for $4.* This clause structure is complex because ELLs have to understand that Irene would receive $4 per car wash and that she only washes one car; this is never stated in the problem but is implied in the construction of the clause. The task to complete is given in the clause *Write an equation that can be used to determine Irene's weekly allowance (a) if she has $12 left after washing the car*. This clause gives a command with the verb *write* and what it is that students are supposed to write, *an equation*. Further information is provided about *an equation* with an embedded clause *that can be used to determine Irene's weekly allowance (a) if she has $12 left after washing the car*. We notice here that the variable is provided through the symbol *(a)* referring to *Irene's weekly allowance,* which could present an additional challenge for ELLs. This symbol has to be used in the construction of the equation, as this test may be completed on a computer that would not recognize if another symbol, such as *(x),* were used instead. The conditional clause *if she has $12 left after washing the car* is another important piece of information for students to consider. *If* clauses are very common in mathematics and have been found to cause particular difficulties for ELLs (Fernandes, Anhalt, & Civil, 2009; Martiniello, 2008). Making sense of the phrase *$12 left* includes understanding that the word *left* means remaining and understanding that the equation does not necessarily require a subtraction. For example, a student could write the equation $\frac{1}{2}a + 4 = 12$ directly if he or she interprets $\frac{1}{2}a$ as already representing what is *left*.

Students may start to symbolize this problem by writing "$a =$…" since they are told to *Write an equation that can be used to determine Irene's weekly allowance.* However, determining the right-hand side of this equation involves a potentially complex interaction with the problem's natural language to mentally undo the actions on a, which adds to the complexity of a problem like this. That is, to come up with the equation $a = 2(12-4)$ the learner would need to work with the values in a different way than how they are presented in the problem. In this instance, it may be easier to think of a as part of what is being manipulated in the equation and not the result. This allows for a more direct translation from the natural language to the symbolic form, which lessens the challenges inherent in this translation process for ELLs.

We see in Table 2 that, in Part 1, students are asked to engage mostly in the formulation stage of problem solving to move from the natural language system to the symbolism system. Some interpretation should also be used to check the meaning of the produced equation against the original natural language. The symbol sense for selecting appropriate symbols to use is done, in part, for the learner by telling them to use (a) for allowance. However, to set up the equation, students must fit this symbol into the larger symbolic representation. It is possible that students could determine the value of the allowance by doing mental computations in their head, but this problem is structured in a way that it requires them to be able to symbolize an equation using the letter *a*, so students must draw on multiple symbol sense elements to complete the task.

Analysis of Part 2 of the Task

In Table 3, we continue with a break down of Part 2 of this task.

Table 3
Application of Framework and Linguistic Analysis of Part 2 of the I-STEP Problem

Clause in Problem	**Problem Solving Stage**	**Symbols Involved**	**Linguistic Analysis in relation to symbols sense: What is required to work with the problem?**
This week Irene used her allowance to buy each of her 5 friends a bracelet…	Formulation	5 friends 1 bracelet per friend	• Making meaning of the thing that is being bought versus the recipient of the action (Reading directly it looks like we could be buying friends instead of bracelets) • Linking the natural language expressed through the word *each* with the number 1
…and still had $3 remaining.	Formulation	$3	• Linking the word remaining to an idea that 3 dollars is what is left after spending (i.e., the result of buying bracelets).
Each bracelet cost the same amount of money.	Formulation	Could let b = cost of one bracelet	• Linking the word each to understand that we can select one variable to represent any one of the bracelets
What was the cost of 1 bracelet?	Solving Formulation Solving Interpretation/ Checking	1st solve $\frac{1}{2}a + 4 = 12$ from Part 1 Create equation: $16 - 5b = 3$ Solve: $b = 13/5 = 2.6$	• Linking the word *cost* to the expectation that the result will be an amount of money • Recognizing an action that is not explicitly stated in the problem • Knowing that to find the cost of one bracelet, we have to find the total cost of all five bracelets and then divide. • Understanding that finding weekly allowance is not what the question is asking – need also to find a new amount of money, the cost of one bracelet by subtracting the cost of all bracelets from the total allowance and knowing that there will be $3 left over • Interpreting the equation against the natural language to check the meaning of equation • Interpreting the result 2.6 as $2.60 to connect to the context in the natural language

Part 2 begins by identifying the time of the next situation– *This week*. In the clause, *This week Irene used her allowance to buy each of her 5 friends a bracelet* we see how Irene used her allowance, but the construction in this clause may cause difficulties for ELLs because *each of* *her friends* is put before *a bracelet*. The next sentence, *Each bracelet cost the same amount of money* establishes an important piece of information for students to solve the problem. ELLs have to connect the word *each* with the numerical representation 1, and recognize that the same variable or letter can represent every bracelet. The question *What was the cost of 1 bracelet?* shows what students need to be able to calculate.

In Part 2, students are required to go through multiple problem solving stages, though not all are explicit in the problem itself. This may cause additional difficulties for ELLs. They must first determine, using their equation from the first part, the actual value of a typical week's (and therefore "this week's") allowance. This involves proceeding through the solving stage. Here students must know the order and properties of operations for "undoing" the equation to get *a* by itself. Once the value of *a* is identified to be $16, however, students must know that the letter *a* is no longer necessary in their work. They need to know the meaning of this variable *a* as representing an unknown that, once determined, will not change again. Links may also be made back to natural language if the students try to interpret or check their solution.

Once they have found *a*, students need to be able to find the cost of one bracelet. The directions to *show all work* require the use of symbolization or visual images (i.e., mental computation will not suffice), so students must again engage in the formulation stage. Students may or may not choose to select a symbolic representation for the cost of a bracelet, such as *c* or *b*. Unlike the first part, they are not given directions on how to symbolize here. The symbolic representation $b = \frac{(16-3)}{5}$ can be used to find the solution, so only numbers are involved in the calculation. However, difficulties could arise, especially for ELLs, because the order in which the calculations need to occur is not the same order in which these values appear in the problem. This could potentially be problematic for ELLs, as they would have to figure out the order of the values by understanding the language that is expressing these values.

Implications for Classroom Teachers and Mathematics Teacher Educators

Teachers and teacher educators know well that the complexity of mathematics language presents challenges for *all* learners, and not just ELLs. As the example shows, however, being able to understand how different semiotic systems are used in the construction of a word problem and how to transition among these systems in solving a problem may present additional challenges for ELLs that are important for teachers to understand. There are additional linguistic demands for ELLs that make developing symbol sense and transitioning between the symbolic and natural language more of a challenge, as they learn to filter their existing and developing knowledge of mathematical language through a second natural language (Brown, 2005).

In particular, the symbolic system may cause great confusion for ELLs because of the ways in which it needs to interact directly with the natural language system throughout the problem solving process. Teachers need to be aware of these potential difficulties and provide opportunities for ELLs to engage with natural language and symbols and the links between them in the context of mathematics teaching. In other words, symbol sense cannot be fully developed in absence of natural language; thus, it is not sufficient to allow ELLs to avoid language issues by engaging them in mainly symbolic tasks. If we expect students to know how these semiotic systems interact in the construction of mathematics, they need experiences that help them build understandings of the multiple semiotic systems at work in mathematics word problems.

A major part of meaning making in mathematics word problems is in the connections between natural language and symbolic representations. As we can see in Tables 2 and 3, the formulation and interpretation stages, where these connections are key, do not just appear once at the beginning or end of the problem but repeatedly throughout the whole process. This suggests that teachers need to be aware of the additional challenges that ELLs developing their language proficiency may have throughout the entire problem. It is not just a matter of helping them remove words and create an equation – they need to develop meaning by constantly checking their symbol sense against the meanings in the natural language of the problem.

As teachers, we need to build a better awareness of the additional challenges that ELLs face with word problems and identify ways to help them use the understandings of language and mathematics that they bring to the table to overcome these challenges. It is critical for teachers to make use of ELLs' many existing skills, ideas and strategies. For example, all students bring with them language and cultural resources (Celedón-Pattichis & Ramirez, 2012) which mathematics teachers should use in authentic ways when constructing word problems to motivate interest and build relationships with and among students. Teachers must also be careful not to relate language fluency with academic competence (Cummins, 1981), but instead recognize that ELLs are often able to communicate sophisticated understanding of mathematics using multiple representations and draw on a range of resources to support their learning, including peers, family, and experiences (Aguirre et al., 2012). ELLs should have opportunities to make use of the tools and resources that work well for them as they build meaning from mathematical problems.

One way that is often recommended for helping ELLs build a connection between the natural language and mathematical symbols is to engage them with a third representation, visualizations. For example, in the problem analyzed here, a teacher may help an ELL to visualize the context by drawing five bracelets or five friends with one bracelet each. Students may also visually represent the money spent on bracelets ($16-$3 = $13) with 13 dots on paper, which they may then partition out one at a time to each of the five friends. However, because 13 is not evenly divisible by 5, the answer is not a whole dollar amount and students may find the visual representation to be not useful. In this instance, a visual image may not be sufficient for helping ELLs develop meaning for all word problems, but may help students recognize the need for symbols, demonstrating again the need for development of strong symbol sense to secure ELLs' success in working with real world situations.

The framework presented in this paper can help teachers connect the problem solving stages to the semiotic systems while providing elements of symbol sense that students, in particular ELLs, can develop in order to work with mathematical word problems. This framework was designed and applied to word problems in middle school mathematics where students begin learning algebra. However, the framework can be adapted and used in other grade levels as well. We see this as one tool for helping teachers to think about new ways of helping ELLs work fluently within the multiple semiotic systems of mathematics in productive and meaningful ways.

References

Abedi, J. (2004). The No Child Left Behind Act and English language learners: Assessment and accountability issues. *Educational Researcher, 33*(1), 4–14.

Aguirre, J.M., & Bunch, G.C. (2012). What's language got to do with it? Identifying language demands in mathematics instruction for English language learners. In S. Celedón-Pattichis & N.G. Ramirez (Eds.). *Beyond good teaching: advancing mathematics education for ELLs* (pp. 183-194). Reston, VA: NCTM.

Aguirre, J.M., Turner, E.E., Bartell, T. G., Drake, C., & Foote, M.Q., & McDuffie, A.R. (2012). Analyzing effective mathematics lessons for English learners: A multiple mathematical lens approach. In S. Celedón-Pattichis & N.G. Ramirez (Eds.). *Beyond good teaching: advancing mathematics education for ELLs* (pp. 207-221). Reston, VA: NCTM.

Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics*, *14*(3), 24-35.

Arcavi, A. (2005). Developing and using symbol sense in mathematics. *For the Learning of Mathematics, 25*(2), 42-47.

Brown, C. L. (2005). Equity of literacy-based math performance assessments for English language learners. *Bilingual Research Journal, 29*(2), 337-363.

Carey, D. A., Fennema, E., Carpenter, T. P., & Franks, M. L. (1995). Equity and mathematics education. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 93–125). New York: Cambridge University Press.

Celedón-Pattichis, S. (2003). Constructing meaning: Think-aloud protocols of ELLs on English and Spanish word problems. *Educators for Urban Minorities*, *2*(2), 74-90.

Celedón-Pattichis, S., & Ramirez, N. G. (2012). Elements of an effective mathematics community for ELLs. In S. Celedón-Pattichis & N.G. Ramirez (Eds.), *Beyond good teaching: advancing mathematics education for ELLs* (pp. 47-53) .Reston, VA: NCTM.

Crowhurst, M. (1994). *Language and learning across the curriculum*. Scarborough, Ontario: Allyn and Bacon.

Cummins, J. (1981). Four misconceptions about language proficiency in bilingual education. *NABE Journal, 5*(3), 31-45.

de Oliveira, L. C. (2012b). The language demands of word problems for English language learners. In S. Celedón-Pattichis & N. Ramirez (Eds.), *Beyond good teaching: Advancing mathematics education for ELLs* (pp. 195-205). Reston, VA: National Council of

Teachers of Mathematics.

de Oliveira, L. C., & Cheng, D. (2011). Language and the multisemiotic nature of mathematics. *The Reading Matrix, 11*(3), 255-268.

Drouhard, J.P., & Teppo, A. (2004). Symbols and language. In K. Stacey, H. Chick, & M. Kendal (Eds.), *The future of the teaching and learning of algebra – The 12th ICMI study* (pp. 227-264). Boston / Dordrecht / New York / London: Kluwer Academic Publishers.

Fernandes, A., Anhalt, C., & Civil, M. (2009). Mathematical interviews to assess Latino students. *Teaching Children Mathematics*, *16*(3), 162-169.

Halliday, M. A. K. (1978). *Language as social semiotic*. London: Edward Arnold Publisher, Ltd.

Indiana Department of Education (2012). *ISTEP+ Mathematics Item Sampler.* Retrieved from http://www.doe.in.gov/sites/default/files/assessment/math-grades6-8-item-sampler.pdf

Kenney, R. (2008). The influence of symbols on pre-calculus students' problem solving goals and activities (Doctoral dissertation). Available from Dissertations and Theses database. (UMI No. 3329205).

Kinzel, M. (1999). Understanding algebraic notation from the students' perspective. *Mathematics Teacher, 95*(5), 436-442.

Martiniello, M. (2006, October). Sources of differential item functioning for English learners in word math problems. Paper presented at the annual meeting of the Northeastern Educational Research Association, New York.

Martiniello, M. (2008). Language and the performance of English-language learners in math word problems. *Harvard Educational Review, 78*(2), 333-368.

Moschkovich, J. N. (2007). Examining mathematical discourse practices. *For The Learning of Mathematics*, *27*(1), 24-30.

O'Halloran, K. L. (2000). Classroom discourse in mathematics: A multisemiotic analysis. *Linguistics and Education, 10*(3), 359-388.

O'Halloran, K. L. (2004). Discourses in secondary school mathematics classrooms according to social class and gender. In J. A. Foley (Ed.), *Language, education and discourse: Functional approaches* (pp. 191-225). London: The Continuum International Publishing Group, Ltd.

O'Halloran, K. L. (2005). *Mathematical discourse: Language, symbolism and visual images*. London: The Continuum International Publishing Group, Ltd.

Pierce, R., & Stacey, K. (2001). A framework for algebraic insight. In J. Bobis, B. Perry, M. Mitchelmore (Eds.), *Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia: Vol. 2. Numeracy and Beyond* (pp. 418-425). Sydney: MERCA.

Pierce, R., & Stacey, K. (2002). Algebraic insight: The algebra needed to use computer algebra systems. *Mathematics Teacher*, *95*(8), 622-627.

Pimm, D. (1995). *Symbols and meanings in school mathematics*. London: Routledge.

Ramirez, N. G., & Celedón-Pattichis, S. (2012). Second language development and implications for the mathematics classroom. In S. Celedón-Pattichis & N.G. Ramirez (Eds.), *Beyond good teaching: Advancing mathematics education for ELLs* (pp. 19-37). Reston, VA: NCTM.

Reed, S. K. (1999). *Word problems: Research and curriculum reform*. Mahwah, NJ: Lawrence Erlbaum Associates.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(1), 139-159.

Solano-Flores, G., & Trumbull, E. (2003). Examining language in context: The need for new research and practice paradigms in the testing of English-language learners. *Educational Researcher, 32*(2), 3–13.

Stacey, K. & MacGregor, M. (1999). Ideas about symbolism that students bring to algebra. In Barbra Moses (Ed.), *Algebraic Thinking Grades K-12* (pp. 308-312). Reston, VA: NCTM.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. F. Coxford (Ed.), *The Ideas of Algebra, K-12* (1988 Yearbook of the NCTM) (pp. 8-19). Reston, VA: NCTM.

Wiest, L. (2001). The role of fantasy contexts in word problems. *Mathematics Education Research Journal, 13(*2), 74-90

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1) How do natural language, symbol sense, and visual representation relate to equity and excellence for ELLs?
2) Consider the test item used in the article. Would you consider this item reasonable for 7th grade ELLs to know how to solve? Why or why not?
3) What does Table 1 reveal about the complexities of mathematics learning for ELLs?
4) How can we best prepare teachers to consider the multiple semiotic systems described in the article to address the needs of their current or future ELLs?

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