STUDENTS' USES AND INTERPRETATIONS OF SYMBOLS WHEN SOLVING PROBLEMS WITH AND WITHOUT A GRAPHING CALCULATOR

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Purpose and Background

As students interact with graphing calculators to help solve math problems, one aspect that they need to attend to is the use of mathematical symbols, which are the components of mathematics that enable communication of solutions and ideas. However, the symbolic language in mathematics is often very confusing for students (Rubenstein & Thompson, 2001). This language barrier may be one reason that students turn to graphing calculators for assistance, but, most non-computer algebra system (CAS) graphing calculators cannot algebraically manipulate symbolic equations to produce useful results.

Studies have shown that students can perform as well in mathematics using graphing calculators as they can without them, but many instructors remain reluctant to teach or assess with this tool (Ellington, 2003). It is reasonable to assume that students who have learned to do mathematics using graphing calculators in high school will continue to use them on homework in college; however, a test that does not permit calculator use may require students to redefine their goal to include communicating a correct solution method for a problem using mathematical symbols. The activity engaged in during homework will no longer be helpful if students have not abstracted a relationship between their activities with the calculator and the symbolic representation of the mathematical concepts involved.

The intent of this research is to conduct a case study of precalculus college students with a focus on their mathematical thinking, particularly about symbols, as they solve problems with and without the assistance of graphing calculators. The question to be addressed is: What is the nature of the goals, activities, and symbolic communication that students engage in when doing work in a practice environment with access to a non-CAS graphing calculator and in an assessment environment without such access?

Framework

In order to address the research questions in this study, a theoretical framework is needed that will provide a lens for looking both at students' anticipation and reflection on goals and activities and the symbol sense that they exhibit as they work with the graphing calculator. The chosen framework is a combination of Simon, Tzur, Heinz, and Kinzel's (2004) reflection on activity-effect relationship framework (AER), and Pierce and Stacey's (2001) framework for algebraic insight. Simon et al.'s (2004) AER framework is built on Piaget's notion of reflective abstraction, and is designed as a way for explaining the development of new mathematical conceptions beyond those already available. The authors describe it as a theory to guide the teaching of mathematical concepts and the design of instructional interventions to address problems in learning mathematics (Simon et al., 2004; Tzur & Simon, 2004). It begins with a goal-directed mental activity, where the learner continually monitors the effects and results of the activity. The learner creates mental records of the relationships between each execution of the activity and the effect produced. By reflecting on these records and looking for patterns between the activities and their effects, the learner abstracts a new activity-effect relationship, which is the basis for a more advanced conception. As a tool for looking at learning, the AER framework can help in the identification of stages of learners' concept development, particularly whether students are at a *participatory* or *anticipatory* stage (Tzur & Simon, 2004).

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Learners' goals and reflections in the AER mechanism are not always conscious to the learner (Simon et al, 2004) and will be difficult to observe. Thus, the algebraic insight framework will be incorporated to evaluate students' understanding when working with technology to solve mathematical problems (Pierce & Stacey, 2001). Algebraic insight is the subset of symbol sense that enables a learner to interact effectively with a CAS when solving problems, and is comprised of two components: algebraic expectation and the ability to link representations. For this study, algebraic insight will be applied to students use of non-CAS graphing calculators to try to solve and manipulate symbolic problems.

Methods

This is a qualitative, multi-case study. The case is defined as an undergraduate precalculus student who frequently uses a graphing calculator to assist in problem solving, thus each of the six different participants constitutes a separate case. The data collection took place during a summer semester at a university in the southern United States, and occurred in the following manner: The research began with individual task-based interviews with each of the students to discuss their experiences with using graphing calculators and solving mathematical problems. During the interview, students were asked to work on three to five algebraic tasks and were prompted to "think out loud" as they solved the problem and/or made use of the graphing calculator. This initial interview was followed by two group sessions where two students worked together on web-based homework assignments for their class. Students had access to textbooks, notes, graphing calculators, and each other in these sessions. A final piece of data came from students' work on their course tests. Students took the test in the classroom where the teacher does not allow calculators, but met soon after the test with the researcher to discuss the goals and activities in which they engaged on the test. The researcher then showed clips of the students' work in the homework sessions and the researcher and participant discussed similarities and differences between the approaches that students took in the practice and assessment environments.

All work in the initial interview, homework sessions, and final interviews was videotaped, and calculator keystrokes were recorded using computer software. These recording techniques where particularly useful in the homework sessions because they allowed the researcher to create a natural work setting for observing students' work with minimal interference (Berry, Graham, & Smith, 2005). The focus of the homework sessions and tests was on quadratic, general polynomial, and rational functions.

Results

At the time that this paper was written, the data collection was still in progress and the analysis was incomplete. Detailed results are to be presented at the PMENA conference, but some initial ideas and observations are shared here. For example, during the initial interviews, the researcher was surprised to find that none of the six participants were able to solve the algebra problem: Solve for *x,* if $x^3+2x-4=8$, by using the graphing features of the calculator, although most of the students had indicated on a survey given the first day of class that they could use the calculator for such a purpose. However, the students were fairly dependent on the graphing calculator for most basic calculations and for checking over any work they did by hand.

The students were all attentive to the fact that the calculators were not going to be permitted on their course tests, and so in the initial interview and homework sessions, most of them insisted that it was best for them to work the problem by hand so that they understood it in the way that would be expected of them on the test. However, when they became stumped, many of them turned to the calculator for help, although not always with a clear purpose in mind of how the calculator was going to be able to assist them. They all had different levels of comfort and trust when using the calculator, and different ideas about how it could be useful to them. For example, one student insisted that the calculator could not help her with any problem that involved a variable, unless it was to allow her to test a value in the expression, while another student went so far as to type an expression like $(x-16)/(x^2+3x-12)$ into the regular calculator screen (i.e., not the graphing screen) in order to "see what it says."

Initial observations about the participants symbol sense leads the researcher to think that they were all fairly weak in this area. For example, one question on the homework asked to students to come up with the quadratic function whose graph was given. Two students followed a similar example in the textbook that used the alternate form of a quadratic function, $f(x) = a(x-h)^2+k$, where (h,k), the vertex, was given on the graph. The students had trouble distinguishing between letters as parameters and variables, and one student did not believe that the final answer could stay in this form, and struggled to get it in the form $f(x) = ax^2+bx+c$ before submitting the problem. All of the students used sloppy notation in their notes, and several made up their own notations to keep track of information in a problem.

These and other observations will be analyzed under the lens of the theoretical framework for this study and results will be shared at the PMENA conference.

Significance

With this study, the researcher hopes to contribute to the understandings of ways that students are intending to use graphing calculators to practice mathematical problems. This may, in turn, identify reasons why students who work successfully with a calculator on homework problems cannot perform well on tests without a calculator. The researcher is not necessarily looking for improvement in test scores, as the intent of this research is not to show that graphing calculator use is superior to pencil and paper methods alone, but will instead be looking for examples of algebraic insight and instances of reflections on the activity-effect relationships, and how the relationships developed with calculator use may have affected the ways that students solved similar problems without a graphing calculator. Attention will focus on how students choose to write up the solution to a problem that they solve with the graphing calculator, and whether they are attentive to the need to clearly explain their work using mathematical symbols.

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