Reflecting to learn mathematics: Supporting PSMTs' pedagogical content knowledge with reflection on writing prompts in mathematics education

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Background

Writing is an uncommon activity in mathematics classrooms. This is particularly true in introductory university-level mathematics courses, where the traditional lecture format remains the most common form of instruction. Many students – including preservice mathematics teachers (PSMTs)– have little exposure to the use of writing to develop mathematical understanding. Writing has been recommended, however, as a tool for both developing and assessing deeper understandings of mathematics (National Council for Teachers of Mathematics, 2000). Providing experiences with writing and opportunities to reflect individually and collaboratively on the these writings is critical for helping PSMTs recognize the benefits of writing a learning tool in the mathematics classroom (Quinn & Wilson, 1997).

For this study, we focus on the use of both writing and reflection in a unique college-teaching seminar designed to support the development of PSMTs' knowledge about both mathematics and pedagogy. This seminar allows PSMTs to teach college-level mathematics while enhancing their own understanding of mathematics and specific pedagogical approaches for the mathematics classroom. Within this seminar, we engaged PSMTs in several activities integrating reflective practice and writing to learn mathematics (WTLM) in order to challenge their current thinking on mathematical topics and enhance their content and pedagogical content knowledge (Shulman, 1987). In so doing, we addressed the question: How does engagement in

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reflective practice and WTLM contribute to preservice mathematics teachers' understanding of what it means to teach and learn mathematics?

The role of reflection

As defined by Dewey (1960), reflection is an "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends" (p. 9). For Dewey, reflection addresses self-identified issues of concern or interest as well as potential explanations for and solutions to these issues. Interaction with others is also necessary in reflection, since an individual needs access to different perspectives and alternative possibilities in order to recognize "the possibility of error" (p. 30) in personal beliefs and understandings. Such reflective collaboration is supported through the attitude Dewey identified as open-mindedness –remaining open to new ideas to alter personally held beliefs and understandings. In this way, previous ways of thinking can be transformed into new understandings that support positive changes in beliefs or actions (Shoffner, 2008).

Through personal experiences and subsequent understandings, teachers continuously create their own practical theory (Handal & Lauvas, 1987). This way of understanding the world guides actions and informs beliefs, which in turn supports how teachers make meaning from new concepts and events. A teacher's experiences, therefore, should be (re)considered through reflective practice in order to address individual beliefs and personal knowledge informing and guiding those experiences (Nilsson, 2008; Shoffner, 2008). Collaborative reflection supports reflective consideration by affirming the value of individual experiences, offering alternative meanings and supporting the process of inquiry needed for reflective practice (Rodgers, 2000).

Reflection can support PSMTs' understandings of teaching mathematics by prompting the consideration of prior knowledge, past experiences and current beliefs (McDuffie & Slavit, 2003; McNaught, 2010; Stockero, 2008a, 2008b). Reflection can also support a teachers' willingness to consider the implementation of new practices in the classroom (Foss, 2010; McGlynn-Stewart, 2010). In this way, reflective practice provides opportunities to consider issues of teaching and learning while fostering the knowledge needed to inform classroom decisions (Shoffner, 2008, 2009a, 2009b; Spalding & Wilson, 2002). In addition, reflection supports multiple ways of making sense of mathematics, helping teachers to become aware of their own thought processes and their students' (mis)understandings.

The consideration and clarification supported through reflection can assist in addressing PSMTs' issues with content and pedagogy. PSMTs frequently fail to understand the difficulties of instruction in their content area despite years of seeing and experiencing classroom teaching. As explained by Nathan and Petrosino (2003), "teachers' subject-matter expertise often overshadows their pedagogical knowledge about how their novice students learn and develop intellectually in the domain of interest" (p. 906). In effect, PSMTs may fail to recognize students' misunderstandings of mathematical concepts that they have mastered due to what Nathan and Petrosino have termed an *expert blind spot* regarding such content knowledge. While preservice secondary mathematics teachers may be not yet be considered "experts" in their field, they do spend much of their class time at the university focusing on upper lever mathematics courses, which does not always lead them to make connections to the K-12 mathematics that they will eventually teach. Given the emphasis for a focus on developing student conceptual understanding and sense making in mathematics, there is a critical need to understand and support effective practices that PSMTs can engage in to overcome their expert blind spots and enhance their content and pedagogical content knowledge in secondary mathematics.

PSMTs need opportunities to reflectively engage with mathematics in ways that perturb and challenge their current understandings. Such reflection is a particularly important aspect of teacher preparation, since "the viewpoints and understandings [they] maintain during their preparation will guide their future actions and beliefs as teachers" (Shoffner, 2008, p. 132). Integrating reflection in WTLM tasks offers a means to expand content knowledge and determine the extent of PSMTs' understandings of mathematics. This knowledge may also be used to alter pedagogical practices for student learning, to overcome practical knowledge arising from the expert blind spot (Nathan & Petrosino, 2003), and to enhance PSMTs' understanding of what is needed for both teaching and learning mathematics.

The role of writing to learn mathematics

Questions that lead learners to make connections and explain reasons behind procedures are critical for developing conceptual mathematical knowledge. WTLM incorporates such questions in the form of writing prompts requiring explanations of mathematical content or processes. For example: Why do we invert and multiply when dividing by a fraction? How do you know that 1/4 is greater than 1/5? How would you explain slope to a friend who has never heard of it?

By responding to writing prompts, learners can monitor and reflect on the strategies and processes used in problem solving (Countryman, 1992; Urquhart, 2009). Writing activities can help learners make connections and apply complex mathematical models and concepts that validate their ideas and foster interconnections between prior knowledge and novel information (Inoue & Buczynski, 2011). Writing can also have a significant impact on learners' abilities to process and retain comprehensive mathematical knowledge, and has been identified as a useful vehicle for developing critical problem solving skills and conceptual understanding of complex ideas with college students (Hein, 1999).

Researchers have also found that WTLM plays a role in advancing learning and assessment of learning (Adu-Gyamfi, Bosse, Faulconer, 2010; Inoue & Buczynski, 2011; McIntosh & Draper, 2001; Miller, 1992; Porter & Masingila, 2000). When integrated effectively into mathematics classrooms, WTLM tasks can help educators understand and assess students' conceptions and knowledge of mathematics (Miller, 1992). Teachers have effectively used writing in their classrooms as a tool for assessing student learning and developing conceptual understanding in their students (Adu-Gyamfi, Bosse, Faulconer, 2010; Quinn & Wilson, 1997).

In this study, WTLM exists in the form of specific prompts asking PSMTs to elaborate on a mathematical topic by writing about it. The prompts require both reflection on and explanations of specific topics and issues in mathematics. For example, the prompt "Explain in writing how you know that 1/4 is bigger than 1/5" encourages PSMTs to consider a known fact that they have come to accept without question is true. This type of reflection on mathematical knowledge is critical for helping teachers consider what is needed for novice learners to develop their own mathematical understanding of such a topic.

Conceptual framework

This study is grounded within a constructivist framework with pedagogical content knowledge (PCK) serving as the focusing lens for analysis. As proposed by Nilsson (2008), "knowledge of subject matter, pedagogy, and context, whether developed separately or integrated, are transformed into a new form of knowledge [(i.e., PCK)] that is more powerful than its constituent parts" (p. 1283). PCK provides a knowledge base for teaching subject matter as well as the means for representing and communicating that subject matter (Shulman, 1987). The ability to understand both content and the representation of content to others is a necessary skill for teachers, particularly when that understanding supports teachers' abilities to organize and adapt content to learners' diverse interests and abilities. By using activities specifically designed to support teachers' development of and reflection on PCK, PSMTs have the opportunity to engage in pedagogically appropriate instructional practices that address the abilities and backgrounds of diverse students.

Chick, Baker, Pham and Cheng (2006) have developed a PCK framework for mathematics that incorporates connections between content and pedagogical knowledge. This framework organizes key elements of PCK into three categories. Category 1, "clearly PCK," identifies moments when pedagogy and content interact inseparably, such as when the teacher discusses student misconceptions about a concept, identifies aspects of the task that affect its complexity or uses an example that highlights a concept or procedure. Category 2 is "content knowledge in a pedagogical context," where the focus is on mathematics content used for teaching. This is evident, for example, when the teacher exhibits thorough conceptual understanding of identified aspects of mathematics or identifies critical mathematical components within a concept that are fundamental for understanding. "Pedagogical knowledge in a content context" is Category 3, which considers generic teaching knowledge used specifically in mathematics teaching. In this instance, a teacher may describe goals for student learning, discuss strategies for engaging students or use generic classroom practices relevant to mathematics learning. We have adapted Chick et al.'s framework (see Table 1) to include examples of teachers' engagement in a PCK category that we felt could be evident in our study.

Table 1. PCK framework (adapted from Chick et al., 2006)

PCK Category	**Evident when the teacher ...**
Clearly PCK	• Discusses general or specific strategies or approaches for teaching a mathematical concept or skill • Discusses student ways of thinking about a concept

	• Discusses student misconceptions about a concept • Identifies aspects of the task that affect its complexity • Discusses how topics fit into the curriculum • Discusses reasons for content being included in the curriculum
Content Knowledge in a Pedagogical Context	• Exhibits conceptual understanding of identified aspects of mathematics • Identifies critical mathematical components within a concept that are fundamental for understanding • Makes connections between concepts and topics
Pedagogical Knowledge in a Content Context	• Describes a goal for students' learning • Discusses or uses strategies for engaging students • Discusses or uses generic classroom practices

In this study, we explore the potential for reflection in WTLM to serve as a useful tool for perturbing PSMTs' existing ways of thinking and enhancing both content knowledge and elements of PCK. It is important to note that it is not our intention to make claims that engagement in reflection on writing is solely responsible for the development of PCK. We are interested, however, in exploring how reflection in WTLM can contribute to PSMTs' pedagogical and mathematical learning, and we use this PCK framework as a way to focus our attention in the data analysis.

Methods

Participants and setting

The participants in the study were seven undergraduate PSMTs enrolled in a unique mathematics course at a research university in the Midwestern United States. Through this course, each

PSMT was assigned to teach his or her own section of College Algebra at the university; in addition, the PSMTs attended a seminar at the end of each teaching day to discuss pedagogical issues, consider mathematical content issues and prepare for the next class. The PSMTs had to apply to be enrolled in this elective course and were selected by their grade point average in their mathematics coursework (must be above 2.8/4.0) and their year in the program (must be at the junior or senior level). Our sample for this study consisted of everyone enrolled in the course. While this represents a convenience sample, the participants do not differ significantly from the extended population of junior or senior mathematics teachers at the same university (who must maintain a 2.5/4.0 GPA in mathematics coursework).

Dr. Kenney (author) served as the instructor for this seminar; during the semester, she led reflective discussions about pedagogy and mathematics, engaged the PSMTs in practice teaching exercises, and observed them in their classrooms. A separate course coordinator oversaw the College Algebra courses and designed the syllabus, pacing guide, common exams and online homework sets. Thus, the only added responsibilities for the undergraduate PSMTs were planning their lessons and creating and grading their own quizzes twice a week. Typical issues that often accompany student teaching in a K-12 setting (e.g., major discipline issues, interruptions in daily schedules, meetings with parents) were absent from this setting, allowing the PSMTs to focus their first teaching experience primarily on the teaching and learning of mathematics. We found this seminar and teaching experience to be an ideal setting for developing and studying PCK.

Drawing from the research literature, opportunities for individual and collaborative reflection were provided in the seminar throughout the semester, including free-writes, group work, class discussions, online discussions and WTLM prompts (described below). These activities encouraged PSMTs to draw on personal understandings of and experiences with mathematics in order to engage in reflective consideration of the topics. In class, for example, discussion often began with PSMTs sharing a pedagogical issue from the college algebra teaching experience and continued with collaborative consideration of why certain practices were or were not productive for student learning.

Data collection

The reflective WTLM prompts used in the seminar were developed by our research team or were adapted from discussion questions in college algebra texts (e.g., Sullivan & Sullivan, 2004) to address the college algebra curriculum. It is important to note that these questions were designed based on the content and not on the PCK framework. They were purposefully designed to be general in scope to elicit students thinking without influencing them to focus on predetermined elements of PCK. The following five prompts were used over the semester, approximately one week before each corresponding mathematical topic was taught in the college algebra course:

PROMPT 1: Explain the difference between the directions “Solve, Evaluate, and Simplify” in math problems. Then write an example using each with the expression 3(x+2)-x.

PROMPT 2: Explain why absolute value problems have two solutions.

PROMPT 3: Given any function $y = f(x)$, explain in your own words why graphing $y = f(x) +2$ will shift this graph UP 2 units, but graphing $y = f(x +2)$ will shift the graph LEFT 2 units.

PROMPT 4: Explain why the graph of a function and its inverse are symmetric around the line $y=x$.

PROMPT 5: What, in your words, is a logarithm? Write a few sentences explaining them to a friend who knows nothing about them.

After writing a response to a prompt, the PSMTs then engaged in collaborative reflection on this learning experience through asynchronous web discussions on a course wiki, posting their own reflection and also responding to several others' posts (c.f., Shoffner, 2008, 2009a, 2009b). The directions given on the wiki were simple: "Reflect on what you learned from doing this prompt yourself. Read and respond to your classmates' reflections too." No specific guidelines were given for what a reflection should involve, other than some in-class discussion of the difference between reflecting and simply recollecting/recalling information.

The teachers were next asked to create a quiz for their students that included the same WTLM prompt they had answered and to post another reflection on the course wiki after reading their college algebra students' responses. The directions on the wiki here stated: "Reflect on what you learned from this exercise. What did you learn about students' thinking? What new ideas did you add to your own mathematics or pedagogical knowledge base? Read and responds to your classmates' reflections." These reflection prompts were phrased generally but focused on individual understanding, allowing the PSMTs to draw on their personal beliefs and experiences before engaging in collaborative consideration.

This process was repeated for each of the five WTLM prompts, focusing on different mathematical concepts being taught in the college algebra classroom. This research design allowed for multiple iterations of reflection on a mathematical concept as the teachers engaged in initial reflection on mathematics by answering each prompt, and further collaborative reflection on their and their students' responses.

Data analysis

PSMTs' reflections served as the primary data source for this study and were analyzed using content analysis to identify patterns in responses (Creswell, 2007). Initially, two researchers used the PCK framework to identify reflection statements that pertained to the different categories. All three researchers then reviewed the initial analysis to come to an agreement on the analysis and to discuss how to share the findings. Because the PCK framework contains so many categories, and because some reflection data did not easily fit into the framework, we then chose to look across the analysis to develop other ways to organize and report the findings. This has resulted in the development of new organizational categories:

- Reflection on self-efficacy
  - Building confidence
  - Dealing with misconceptions
- Reflection on student thinking.
  - Understanding students' mathematical conceptions
  - Articulating mathematical reasoning

These categories serve as an outline for the findings section below where they are used to present the findings in relation to the PCK framework

Findings

In each subsection below, we share direct quotes from the PSMTs' reflections as illustrations of ways in which their thinking about the content and pedagogy surrounding the WTLM prompts elicited elements of PCK.

Reflections on self-efficacy

Building confidence

The PSMTs' reflections on their responses to the writing prompts showed an acknowledgment of their own gaps in mathematical content knowledge, evidence of attending to content knowledge in a pedagogical context (Category 2 in the PCK framework). However, rather than exhibiting their knowledge, the PSMTs focused on their self-doubts or limitations in knowledge. In response to Prompt 1, for example, six out of the seven PSMTs struggled with seeing a difference between the terms evaluate and solve, with a few initially seeing these terms as interchangeable. One PSMT opened her reflection by saying that the prompt took her out of her "comfort zone." She expounded,

> I should probably learn the difference in [these terms] so that I can know which language to use in different problems. I guess it's also a little annoying/frustrating to me because one of the reasons I like math is because I don't often have to focus on my language… But to be a good teacher, I know I need to be universally good in both language and numbers.

Another PSMT reiterated this sentiment, stating, "When I finished this problem I had to laugh. I cannot remember the last time I struggled so much with defining and explaining why these words were different from each other." Their reflections emphasized the perturbations this prompt caused in PSMT thinking about their own content knowledge. From their responses, they were uncomfortable coming to terms with their knowledge deficits. However, they accepted the fact that they would have to overcome these deficits as a teacher. As one PSMT explained, "This prompt brings to light how many mathematical ideas I probably don't completely understand…I will probably be spending a lot of time in the future relearning the things that I ignored when I was originally taught them."

Dealing with misunderstanding

PSMTs also began to recognize their individual issues with pedagogy and content, encapsulated by the expert blind spot. They commented that, although they thought certain facts were very easy, they noticed that their college algebra students did not always "get the whole picture." For example, in response to Prompt 4, one PSMT was surprised that, although his students understood that a function and its inverse were reflected about the line $y = x$, "this understanding doesn't come naturally with some notion that there are certain functions that it wouldn't be natural to do this with.” This PSMT was clearly identifying critical mathematical components within a concept that were fundamental for his students’ understanding, an example of content knowledge in a pedagogical context (Category 2). Another PSMT made a clear connection between her own struggles and those of her students, explaining:

> Grading my quizzes today, I realized that a lot of students tried to factor when the problem was to expand and a lot of other students expanded when they were told to factor. I think if I had not done this prompt before grading, I might have been a little more judgmental. I probably would have thought: Can't these kids read directions? But after doing the prompt I can now think of it in a different way…it is not necessarily the student's fault for doing the problem but maybe it’s the fault of the way teachers teach mathematics.

In relation to the framework, this PSMT identified aspects of the task that affected its complexity as a result of her work on the WTLM prompts, which is an example in the framework under Clearly PCK (Category 1). She is exhibiting a change in the way she thinks about how the representation of content to others can affect learning and how she may need to adapt to learners’ needs as a teacher. Another PSMT responded to the reflection above by positing, “When we realize that we too struggle it is easier to understand the difficulties that our students go through.” Thus, the prompts helped these PSMTs to gain awareness of the need to close the gap between the way they think about mathematics and the way their students come to know mathematical ideas.

The PSMTs’ reflections also revealed that completing the prompts encouraged them to consider their own teaching practices and the practices under which they learned mathematics. As one PSMT reflected, “I feel like we’re so focused on the math and the concepts that we don’t look closely enough at what everything really means. We should put more effort into complete understanding rather than getting all the basics down." A second PSMT, focusing on Prompt 3, explained:

> Thinking of the original points as $(x, f(x))$ and the new points of the new graph as $(x, f(x) + 2)$ makes a lot of sense to me…However, I am not sure this is the best way to explain this to people who are learning this for the first time. This is where a concrete example would really help a lot I believe.

Statements such as these exemplify PCK through PSMTs' discussion of strategies or approaches for teaching a mathematical concept (Category 1). Their reflections on these WTLM tasks supported their consideration of their beliefs about what constitutes effective teaching strategies for meeting learners' needs.

Prompt 1 was particularly powerful in raising PSMTs' awareness of the impact of the terminology they use in class. For example, one PSMT explained,

> Students can sometimes become confused if we use the wrong terminology or use the same terminology for two separate things…Hopefully, I will be able to keep an eye on my terminology and be able to communicate with my students efficiently and effectively.

In a response to this reflection, a second PSMT reframed this issues by stating, "We must make sure as teachers that we not only choose words carefully, but also consider the context in which the words are being used." A third continued, "I realized then that it's no wonder students always get confused in what we want them to do…By using the correct mathematical language, we are able to help students understand exactly what we are asking of them." This prompt supported the PSMTs' development of pedagogical knowledge in a content context (Category 3) by considering the role of language as an instructional strategy for engaging students in the classroom. We see again a way in which the teachers' ways of understanding the importance of how content is represented to others is changing to include a consideration of others' perspectives and not just their own. This is a crucial move in preparing teachers to present and adapt content to diverse groups of learners at all levels.

Reflection on student thinking

Understanding students' mathematical conceptions

For many PSMTs, using the same prompts they had answered with their students allowed them to consider mathematical issues from different perspectives. As one PSMT explained, many of her students set the given equation equal to zero when defining the term solve in Prompt 1, just as she did in her own response. She shared:

> Another thing that I noticed, I did it in my own writing as well, was that when solving, the students set the equation equal to zero, even though it was written without an equal sign. Is this a safe assumption to make, or did we completely change the meaning of the problem? Were we supposed to solve the problem without finding a numeric solution? Whatever the answer, it's really interesting that just about everyone set the equation equal to zero even though the directions never said to do so.

A second PSMT responded to this reflection, noting that at least one of her students had noticed that the problem was not an equation: "One student actually said we could not solve the problem since it was not set equal to anything, and I thought that was an interesting take on it. It was definitely not something I had thought about." For these PSMTs, attending to their students' conceptions of the problem alerted them to components of the mathematical problem that they had not yet considered, an element of PCK (Category 1). We would also suggest that this prompt encouraged the PSMTs to think about deep conceptual understandings of concepts that they had not come to terms with themselves (Category 2).

Articulating mathematical reasoning

Several instances of the PCK framework component, considering students' ways of thinking about a concept, were evident in the reflections. In many cases, the PSMTs attributed a higher level of understanding to the students than was evident in their responses. For example, one PSMT discussed responses to Prompt 4 from her class, sharing,

> Some seemed to understand in class, but when they were asked to write why on a piece of paper, they wrote, "I have no idea why" or "because that's what the numbers do." Overall, I think the majority of my class knows what to do when they see those numbers, but they aren't exactly sure how to explain it or put it into words.

The idea that one could understand the concept but not be able to explain it "in words" was also a common explanation provided by the PSMTs as a reason for why they were not able to write easily about the given prompt. As the PSMTs and their students were not practiced in writing about mathematics, they may not have had the opportunity to develop the vocabulary for putting their understanding into words. However, the PSMTs appeared to struggle with the requisite ability to identify differences in content knowledge (Category 2) between having the skills for solving mathematical problems and exhibiting deep conceptual understanding of the problems.

PSMTs often attempted to interpret students' thinking, especially when the students had stated something incorrectly. For example, one PSMT explained,

> I had one student say that a non one-to-one function can't have an inverse because the original can't be graphed. This is obviously wrong, but I think I may have understood where he was going with his answer. I think he meant that were the inverse of a non one-to-one function to exist, it wouldn't pass the vertical line test and thus not be a function.

In many cases, discussions of students' responses were accompanied by rationalizations of what the students may have "really" meant. Developing their own interpretations of students' responses is an important component of PCK Category 1 element of discussing student ways of thinking about a concept. However, it is possible that making incorrect assumptions about what students are understanding can lead teachers to move too quickly to the next topic in class.

Finally, when discussing their students' responses, the PSMTs often came to realizations of students' typical levels of understanding. Prompts 2 and 5 were particularly helpful for the PSMTs to understand some student habits exhibited in their classroom. For example, after reading students' responses to Prompt 2, a PSMT concluded,

> Most students know what the absolute value bars "do" to a problem but it is the reason why this happens that escapes them. This makes me think that the students are just focusing on how to solve a certain problem in front of them, plug in an answer, and not think anymore about the problem.

Similarly, after recognizing his own words used in class in students' descriptions of what a logarithm was, a PSMT mused, "I am almost learning that sometimes students will just reiterate what you are telling them like they were parrots." A second PSMT made a similar observation, sharing, "I have found that these prompts normally bring out what the students heard me say in class. They really do just regurgitate what I say sometimes without questioning my reason." For experienced teachers, these revelations may seem obvious, but for novice teachers, this understanding was both eye opening and an important learning experience and serves as evidence of growth in their pedagogical knowledge (Category 3).

Conclusions

In order to disrupt the basic schema of "knowing" mathematics, PSMTs need opportunities to engage with mathematics in ways that interrogate and reframe their existing understandings. Reflective practice supports such interrogation and reframing by encouraging the (re)consideration of experiences, knowledge and beliefs. This study shows that engagement in reflective WTLM activities allows PSMTs to examine their views on what it means to teach and understand mathematics while developing their pedagogy in a mathematics classroom. We believe that a particularly important aspect of these activities is the incorporation of multiple layers and iterations of reflection as they write their own WTLM prompt responses, as they reflect on their writing, and as they reflect on their students' responses to the same prompt. We feel this aided their ability to not just recall facts about their learning or teaching, but to include, as Dewey (1960) suggests, potential explanations for and solutions to the issues they identified in their own learning or teaching strategies. In this way, reflection and WTLM serve as tools for PSMTs to think about mathematics in different ways, supporting an expanded view of mathematics that, in turn, may enhance pedagogical content knowledge.

We have previously discussed the role of reflection in helping teachers transform previous ways of thinking into new beliefs that support changes in practice (Shoffner, 2008). Although the participants in our study are novice in their teaching practices, we see evidence in their reflections of these kinds of transformations already taking place. There are multiple examples of the PSMTs recognizing a need to consider mathematics from another person's point of view instead of just their own. A significant change in thinking about teaching is also evident as they came to realizations of students' typical levels of understanding. Issues of ways in which learners may "parrot" what a teacher says or does to complete a math task were discussed in class, but they did not become real or meaningful to the PSMTs until they saw their own words repeated in students' writings. The opportunities provided by the WTLM prompts to consider any past experiences with and current beliefs on teaching and learning mathematics speaks to the importance of reflective practice in addressing and mitigating the expert blind spot.

As van Driel, Beijaard, and Verloop (2001) explain, PCK can be developed through classroom teaching experiences that draw upon subject matter knowledge. We see this seminar and subsequent teaching experience as an ideal setting for developing and studying PCK. Although we do not claim that the PSMTs developed all elements of the PCK framework, we do assert that the examples shared through this study help build ideas about how reflection and WTLM tasks can contribute to PSMTs' learning about their own and their students' mathematical knowledge as well as best practices in teaching. By integrating reflection and WTLM, the PSMTs in this study were able to acknowledge gaps in mathematical content knowledge, identify areas they need to examine in their instruction and recognize different ways that students think about mathematics, all of which are important components for enhancing pedagogical content knowledge.

Development of these components is also important for enhancing teachers' PCK for the K-12 classroom. Differences in classroom context, student populations, teacher responsibilities and curricular expectations are factors to consider when examining the development of teacher reflection and WTLM use in the K-12 setting. Student engagement with algebra, for example, may differ significantly between a university and K-12 classroom due to issues of motivation, experiences in mathematics, and family or cultural influences on learning. While helping PSTs make this connection was beyond the scope of the study, we recognize a need for further research to examine how to support teachers' reflective practice and use of WTLM in K-12 classrooms. As this study shows, teacher engagement in reflection and WTLM can support the development of pedagogical content knowledge, suggesting that additional exploration of these practices can provide insights for teacher researchers and teachers to better understand and support best practices in the classroom at all levels of learning.

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