Abstract

We examine preservice mathematics teachers' conceptions of writing as a tool for learning mathematics before and after participation in and reflection on writing tasks. We describe the use of two targeted activities incorporated into a secondary methods course: writing to learn mathematics (WTLM) and reflection on that writing. Prior to participation in these activities, the preservice teachers expressed reluctance toward the use of writing in mathematics and uncertainty as to how writing could be useful in mathematics, while accepting that some possible benefits might exist for students' procedural learning. Following participation in these activities, the preservice teachers expressed a willingness to accept writing as a useful tool for supporting an expanded view of teaching and learning mathematics. Specifically, the preservice teachers considered writing as a way to build connections between mathematics and other subjects**,** a means to assess student understanding of mathematics, and a beneficial support for student conceptual learning**.**

Keywords: secondary preservice teachers; mathematics; writing to learn mathematics; reflection

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Reflecting on the Use of Writing to Promote Mathematical Learning:

An Examination of Preservice Mathematics Teachers' Perspectives

Rachael H. Kenney, Melanie Shoffner, and David Norris

Introduction

Mathematics and writing are two topics that are not typically associated with one another, especially from students' perspectives: one writes in language or history courses, and one solves math problems in math class. However, writing can be "a fundamental mode of learning" (Stehney, 1990, p. 27) that serves a critical role across all disciplines, including mathematics. Through written expression and explanation, learners can engage with mathematical ideas, content and processes in ways that they may not have done before (Countrymen, 1992; Parsons, 2011). For preservice teachers who are preparing to move from the higher level mathematics course they have been engaged in at a university to work with K-12 students who are learning basic mathematical concepts for the first time, examining the mathematics that they "know" in new ways is crucial to understanding connections between content and pedagogy and developing pedagogical content knowledge (Nathan & Petrosino, 2003; Shulman, 1987).

In this study, writing in the form of a tool referred to as *Writing to Learn Mathematics* (WTLM) was incorporated to support and challenge preservice mathematics teachers' (PSTs') understandings of mathematics and teaching. For example, asking PSTs to reflect in writing to a prompt such as "How do you know that 1/4 is greater than 1/5?" requires an interrogation of their own understanding of mathematical content or processes. In alignment with WTLM, the practice of reflection on writing prompt responses, either those written by others or by oneself, was used to support PSTs' understanding of mathematics content and pedagogy. This article examines the conceptions preservice mathematics teachers hold on the use of writing for learning

mathematics, concentrating on the conceptions held before and after participation in and reflection on writing tasks. We also suggest potential benefits for mathematics teachers from engaging in such tasks in professional development.

Background

Why Introduce WTLM to Preservice Teachers?

Research and practical experiences with teachers suggests that coursework in upper-level mathematics does not automatically encourage PSTs to make connections to novice conceptions held by the K-12 mathematics students (Nathan & Petrosino, 2003). One reason to explicitly introduce activities that challenge the ways preservice teachers think about learning mathematics is to help address what Nathan and Petrosino (2003) term *the expert blind spot*: an inability for teachers to recall the ways in which they first developed conceptions in mathematics. In the classroom, the expert blind spot means that teachers' own understandings of subject matter can overshadow their pedagogical knowledge of what is needed for novice learners to develop their mathematical knowledge. This can lead them to put more focus on efficient and effective (from a teacher's perspective) problem solving procedures and limit reflection on the processes used by novices struggling to make sense of new ideas (Nathan, Koedinger, & Alibabi, 2001). One way to address the expert blind spot is for teacher educators to find ways to perturb PSTs' schema of what it means to learn and know mathematics. In this study, we propose the use of reflection on WTLM to achieve this perturbation.

The Role of WTLM in Teaching and Learning

WTLM involves the use of writing prompts to enhance the understanding and teaching of mathematics, with the prompts requiring reflection on or explanations of topics and issues in the subject area. WTLM prompts can focus on attitudinal or affective issues or require explanations

of mathematical content or processes (e.g. "My best experience in math was when…, What does it mean to say that two equations are equivalent? When I see a word problem, the first thing I do is…?). When used as a teaching tool, the perceived benefits of writing prompts such as these could be divided into three general categories: benefits for the student as writer, benefits to the teacher as reader, and benefits to the student-teacher interaction (Rose, 1989). By writing their own response to the prompts, teachers are challenged to unpack and reflect on their own ways of thinking about mathematical ideas that have become routine for them. Through writing, PSTs can monitor and reflect on the strategies and processes that *they* use in problem solving (Countryman, 1992; Inoue & Buczynski, 2011), that may help them think about the ways that novice learners come to know a mathematical concept. WTLM can also advance learning and assessment of learning (Adu-Gyamfi, Bosse, Faulconer, 2010; McIntosh & Draper, 2001; Porter & Masingila, 2000) by providing teachers and students with agency in their mathematical learning processes. In addition, when integrated effectively into mathematics classrooms, WTLM tasks can help teachers understand and assess students' conceptions and conceptual knowledge of mathematics (Burns, 2004; Miller, 1991; 1992).

Writing about the benefits of WTLM as a doorway to interdisciplinary and intellectual development, Burton (1992) explains, "Expressing ideas explicitly helps students to reflect on the schemas they have constructed and to develop intellectual skills that allow them to approach any discipline intelligently" (p. 28). The experience of writing itself should be of value for the learner, for when the product or outcome of writing takes a second place, learners can focus on the epistemological aspects of the experience (Yagelski, 2012). Hence, they can take their understanding of the mathematical concepts to a level beyond memorization, replication, or mimicking. This is true for PSTs as learners, as well.

Conceptual Framework

This study is grounded in a framework of social constructivism and the conceptual lenses of communities of practice, practical theory, and reflection. As defined by Riel and Fulton (2001), a community of practice is "a group of people who share a common interest in a topic or area as well as a particular way of talking about the phenomena, tools, and sense-making approaches for building their collaborative knowledge with a set of common collective tasks" (p. 519). The collaborative knowledge developed within a community of practice supports the development of what Handal and Lauvas (1987) term practical theory, which is defined as the knowledge, experience and values used by teachers to guide "daily actions and understandings of teaching and learning" (Shoffner, 2008, p. 125). Specific to this project, secondary mathematics PSTs work within a community of practice to extend their practical theory on the use of writing as a tool for mathematical learning.

Reflection can support an understanding of teaching mathematics through the consideration of prior knowledge, past experiences, and current beliefs (McDuffie & Slavit, 2003; McNaught, 2010; Stockero, 2008a, 2008b), which makes it a useful activity to incorporate with writing prompts intended to help PSTs unpack their thinking about mathematical content and pedagogy. As defined by Dewey (1960), reflection is an "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends" (p. 9). The expectation from Dewey is that reflection includes self-identified issues of concern or interest, but also involves suggestions of explanations and solutions to these issues. In this way, one's previous way of thinking can be transformed into new understandings that support change (Shoffner, 2008). When enacted as a deliberate way of thinking that leads to change in action (Shoffner, 2009a), reflection allows

PSTs to consider educational issues from different perspectives while developing and refining the knowledge needed to guide their teaching (Spalding & Wilson, 2002; Zeichner & Liston, 1996). As such, reflective practice has the ability to support PSTs' examinations of teaching mathematics, an important aspect of teacher preparation since PSTs' future actions and beliefs are guided by the understandings they develop during their preparation (Shoffner, 2008). Reflection also provides a means to engage PSTs' existing understandings of mathematical content knowledge while supporting the development of their pedagogical content knowledge as it relates to the teaching and learning of mathematics.

In our research, we hypothesize that incorporating and eventually normalizing the use of writing in mathematics offers multiple benefits to PSTs committed to effective teaching, conscious reflection, and understanding the diverse ways in which students learn, discover, and create (Adu-Gyamfi et al., 2010; Miller, 1992). It is important, however, to first examine secondary PSTs' conceptions of writing as a tool for learning mathematics before and after authentic engagement in writing tasks to understand ways in which teachers may make personal use of such a tool. Thus we see reflection as a key aspect of PSTs' professional development in this study. It can allow PSTs to analyze issues of teaching and learning while fostering the knowledge needed to make classroom decisions (Shoffner, 2008, 2009a, 2009b; Spalding & Wilson, 2002). Reflective practice can be used as one means of supporting the incorporation of WTLM into the mathematics classroom because reflection may help teachers recognize its usefulness as a learning tool (Quinn & Wilson, 1997).

Methods

Participants and Context

To investigate PSTs' perceptions on the use of writing in mathematics, we designed a

qualitative study with 22 PSTs enrolled in a secondary mathematics methods course at a large Midwestern university in the United States. In following with Nathan and Petrosino (2003), our participants were considered to have high subject-matter knowledge because they had completed courses beyond calculus in the university curriculum (most had, in fact, nearly completed the equivalent of an undergraduate degree in mathematics). Eighteen of the participants were seniors at the university, while the remaining four were enrolled in a post-baccalaureate teaching certification program.

The mathematics teaching methods course in which participants were enrolled was taken during their last semester at the university. The course met once a week for six weeks and was immediately followed by ten weeks of a student-teaching internship in local secondary schools. During the six-week methods course, the PSTs were placed in a field experience at the school in which they would ultimately complete their internship and were out in the field for several hours each week. We incorporated five WTLM and reflection tasks to be part of the six course meetings, homework, and field experience activities.

Data Collection

Course wiki. Our data for this study included three course WTLM activities, in-class discussions and reflections, and three asynchronous reflection discussions posted to a course wiki. Our choice of a wiki for reflection and discussion was influenced by studies that have shown the benefit of PSTs' reflection through technology-enhanced learning (Cheung & Foon Hew, 2004). Teaching that incorporates learning logs and asynchronous discussion sessions has been used extensively by scholars working with the development of reflective practice among PSTs (Cheung & Foon Hew, 2004; Shoffner, 2008, 2009a, 2009b). Collecting data from a wiki also ensures that researchers capture an accurate record of the PSTs thoughts. In this study, the

wiki structure permitted the PSTs to easily view and respond to each other's responses. We used a secure system so that data was restricted to only those in the course and the authors. Group pages were created, with three to four PSTs assigned to each group; the PSTs were expected to read and respond only to group members' reflections for each assignment (see Figure 1 for the wiki template used). During the six-week course, we collected 122 reflective posts from the wiki, with an average of five to six posts per participant.

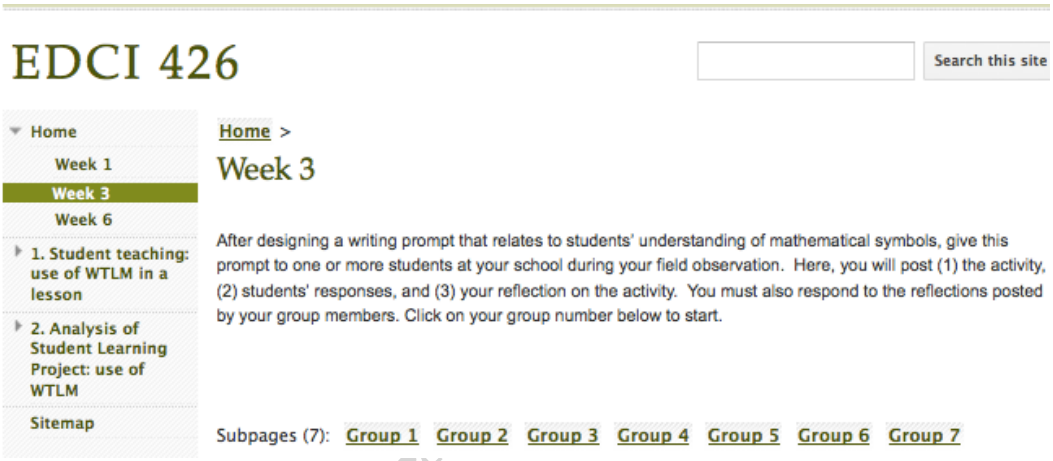

Figure 1. Wiki page used for data collection

WTLM and reflection tasks. In the first week of the methods course, we provided a brief introduction to WTLM and to reflection. We explained our conception of reflection with the PSTs, primarily that reflection involves identifying issues of personal concern or interest, but should also involves suggestions of explanations and solutions to these issues (Dewey, 1960). Discussions about the difference between recollection/recall and reflection also took place. In this first week, we kept the discussion of WTLM brief in order to influence the PSTs' initial perceptions of writing in mathematics as little as possible. The first assignment was then to respond on the wiki to reflection prompts given these three guidelines:

1. Reflect on any previous experiences you have had with writing in your own

mathematics classes;

2. Reflect on the possible role(s) of using writing in mathematics classes; and
3. Reflect on the pros/cons of incorporating writing in mathematical learning.

Because these prompts asked directly for reflection on the use of writing, these wiki posts serve as data in this paper for PSTs' initial conceptions of WTLM.

In weeks two, three and six, we incorporated writing and reflection activities into the course. These activities comprised an "intervention" of sorts to briefly introduce the notion of WTLM to the PSTs. For example, during class in week two, we asked the PSTs to write their own responses to some WTLM prompts (e.g., How do you know that 1/4 is greater than 1/5? Explain your thinking). The PSTs were given several minutes to write their own responses to each prompt before being asked to reflect on the process of completing such prompts and how it helped them think about mathematics. Reflections were written or discussed aloud in groups and recorded.

In the next week, PSTs created their own WTLM prompts and gave them to students at their field placement. After collecting responses from students, PSTs posted the prompt, sample student responses, and a reflection on what they learned from the experience to the course wiki. They also responded to the prompts from others in their wiki group.

In week six, the PSTs considered the use of rubrics for evaluating WTLM assignments. They examined actual student work from a mathematics writing assignment created and shared by a local school mathematics teacher and then designed original rubrics for the assignment. Reflections/discussions on the creation of rubrics and how well they allow one to understand students' thinking and work were recorded.

At the end of week six, PSTs posted a final reflection to the course wiki. The guidelines

for this reflection were simply to "post your reflection concerning the role of WTLM." These posts served as data here for PSTs' developing conceptions on WTLM. We used the term "developing" rather than "final" perspectives with the understanding that six weeks is hardly enough time for our participants to have fully identified with a perspective on this tool. It is important to note that the fact that there were changes in PSTs' conceptions or beliefs about writing in mathematics was not surprising and was not the main finding reported here. Instead, we focused on sharing the preconceived notions of writing in mathematics that PSTs may bring to a methods course and examples of specific ways in which their conceptions can change with a limited amount of exposure to writing tasks. Our intention in sharing our findings is to motivate mathematics educators to see a benefit in considering activities similar to the ones we describe here in teacher professional development as a way of enhancing PCK.

Data Analysis

In this study, we examined the research question: What conceptions do preservice mathematics teachers hold on the use of writing as a tool for learning mathematics before and after authentic participation in and reflection on writing tasks? To address this question, we focused our analysis on the first and last of the tasks described above as they pertained to PSTs' prior and developing conceptions of WTLM. For that reason, a detailed analysis of tasks two thru four is beyond the scope of this report. However, we do share a brief description of ideas produced by PSTs on these tasks to help the reader better understand the study's context.

To analyze the collected data, the reflective posts from weeks one and six were compiled and distributed for analysis to our research team following the conclusion of the course. These reflections served as a primary data source and were analyzed using content analysis to examine the data as a whole and identify patterns in responses (Creswell, 2007; Patton, 2002). To code,

research team members independently conducted an initial analysis of the wiki reflection posts, identifying common and repeating elements of PSTs' perspectives in their reflections. We then met as a group to determine similarities and differences, and refined overarching categories related to the use of WTLM as a learning tool in mathematics. Additional analysis revealed specific common themes within these categories, as discussed in the next section. This analysis provided us with a way to describe and discuss PSTs' perspectives toward WTLM and allowed for identification of representative examples of each theme from the data.

Results and Discussion

PSTs' Initial Perspectives on WTLM

During the six-week course, PSTs shared concerns about and successes with writing as a tool for learning, while engaging in discussions on the course wiki about the varying roles writing could or should play in learning mathematics. From the wiki posts completed in week 1, we identified three main categories of perspectives on writing that we coded in the data: (a) reluctance toward the use of writing in mathematics based on personal experiences, (b) an uncertainty of how writing could be useful for promoting student learning in mathematics, and (c) some acceptance that possible benefits may exist. All 22 PSTs posted some comment that fit into one of these three categories. In the descriptions below, we share examples of the PST's reflections as representations of the initial perspectives they brought to the methods course about the use of writing in mathematics.

Reluctance based on personal experiences. In their initial reflections, we found that PSTs drew on their past personal experiences in mathematics and revealed a common feeling that writing had little place in the mathematics classroom except in specific circumstances. For example, many PSTs could not recall having many (or any) writing opportunities in high school

mathematics classes and felt that it was, perhaps purposefully, "kept to a minimum." Those that did recall writing in mathematics explained that, from their experiences, it was limited to geometry or proofs. As one PST shared, "The largest and most common role that writing can be used for in mathematics is proof writing." Many saw writing as part of the geometry curricula, although they often identified it as disrupting what they were "used to doing" in mathematics. One explained, "When writing proofs was first required, I wasn't always sure how to express in words things that I was accustomed to explaining with mathematical symbols."

PSTs' inexperience with writing in the mathematics classroom supported an initial reluctance toward using it in their future classrooms, influencing their perceptions on how students might view WTLM. For example, many PSTs expressed that their students might not believe that writing belonged in a mathematics classroom; subsequently, their students might not be comfortable using writing as a tool. As one PST explained, "I know a lot of my students will hate writing in math because they don't think that math should be about writing."

Some PSTs were reluctant to expand their definition of what belongs in mathematics or a mathematics classroom, exemplified by the comment, "Math class is supposed to be about mathematics and if students spend too much time writing, then it will become an English class instead." Others were concerned that a lack of experience with writing in mathematics would make students reluctant to embrace it, detracting from students' learning by making them uncomfortable in the mathematics class. One PST explained:

> Students who struggle with writing will then feel uncomfortable in math class. This creates more problems for those students because now, not only do they struggle with writing, but they probably also will struggle in math class. Also, students who enjoy math, but don't enjoy writing might be discouraged.

PSTs also initially expressed concern that "students can get too focused on their writings instead of the math concepts they are really supposed to be learning," illustrating reluctance to expand existing views on the nature of a mathematics class: words are not a part of the mathematics curricula. Many PSTs indicated their agreement with this conclusion, suggesting that writing in a mathematics classroom would be distracting from the "real" mathematics.

Uncertainty with how writing contributes to student learning in mathematics. The PSTs' lack of experience with and understanding of writing in the mathematics classroom also contributed to their initial uncertainty about incorporating WTLM for student learning in their future classrooms. Although reflections in this category did not suggest that they were explicitly against the use of writing, the PSTs expressed uncertainty as to the role WTLM could play in students' mathematical learning and were concerned with the amount of time writing activities would take in the classroom.

Several PSTs explained that they would only be willing to ask their students to use writing in mathematics if it had a "specific purpose" in the curriculum. As a PST shared, "Whenever I had to write, it wasn't relevant to what we were learning. Because of that, I did not take the writing seriously." Another expanded on this sentiment, explaining, "When writing is tacked on as an afterthought...it may play more of a fluff role than deepen learning." Without a clear objective for the writing, PSTs were hesitant to accept WTLM's place in the classroom:

> When writing in mathematics is not completely connected to the goal of their mathematical learning but is instead sloppily incorporated as a side-note, however, it may actually detract from student learning. This was the case in my high school geometry class in which we wrote a research paper. The research paper had little connection to what we were actually learning in class. Instead, it seemed like a futile attempt by the

teacher to incorporate writing.

Such uncertainty over using WTLM to support student learning is understandable given PSTs' stated lack of experiences with the positive integration of writing; without an understanding of how teachers could "use writing at appropriate times," PSTs were concerned that writing would "take away" from students learning of mathematics.

The issue of writing possibly taking away, rather than contributing to student learning, was also addressed through PSTs' concerns with time. Several believed that WTLM activities would take more time than other activities, especially if students struggled with writing, in general; in this situation, students would "spend too much time…figuring out the writing" and therefore "lose a little of the math." Incorporating writing could also take important time away from what teachers are supposed to do, as exemplified by one PST's comment: "I think that a con of incorporating writing in to the classroom [is that it] would take some time away from making sure the students grasp the basic concept of what they are learning."

Acceptance of the possible benefits of writing. Despite their expressed reluctance and uncertainty, several of the PSTs brought to the methods course some initial acceptance that certain types of writing could be useful in some ways in the mathematics classroom. For example, they suggested that if writing was specifically linked to procedural learning, students could have the opportunity to use writing to help memorize steps or organize their thinking. One PST explained that students could "write down the steps for solving a problem, then later go back and be able to follow their steps and successfully solve the problem without being confused." Personal experience again played a role on this view on writing; positive experiences supported several of the PSTs' acceptance of writing's usefulness, as exemplified by the reflection:

> [It can] help students keep a record and they will also be able to know what they were doing. I know if I quickly write something down without actually describing what I am doing I will come to it later and have no idea what I was working on. By writing out what they are doing into complete sentences it helps students keep track of what they were working on and their process of thinking about a certain problem.

These PSTs referred to improvements in procedural understanding that could be gained from writing, but only a few made any mention initially of specific ways in which it could help students develop conceptual understanding.

A few PSTs initially suggested that WTLM also held potential benefits for the teacher. As one PST reflected, "Teachers do not always know what each student thinks on a problem. Writing can help the teacher to achieve that, and it can also help the teacher to know how each student learns differently in order to adjust teaching methods." From this viewpoint, PSTs did share preconceptions that writing in the mathematics classroom could be useful for making students' thinking accessible to the teacher.

Reflections in the "Intervention" Activities

In weeks 2 and 3 of the course, we used three activities to increase opportunities for PSTs to consider the use of WTLM. As mentioned above, we have not made the reflections on these activities a major focus of this article because the majority of reflective posts dealt more with the PSTs own mathematical knowledge instead of conceptions of the use of writing in the mathematics classroom. However, some discussion was included that related to the PSTs' developing conceptions of the use of WLTM. We share a brief summary of these ideas and a sample of reflection posts from these "intervention" activities to help the reader better understand the context of the experiences gained in the methods course that may contribute to

answering our research question (see Table 1).

Table 1.

Reflections from Weeks 2, 3, and 6 "Intervention" Tasks.

WTLM Task	Reflections
Week 2: PSTs answered the WTLM prompt, Write a note explaining to a younger student what you need to *know* and *do* to factor $8x^2 - 2x - 21$ and reflected on the activity.	• Writing an explanation makes you realize…it is necessary for a teacher to know how to explain using correct language instead of just carrying out a procedure • I never thought about how to teach the FOIL method before – I have always been accustomed to just saying something like, "just FOIL it." • It is extremely hard to explain the basics of such a topic when it has become such a rote routine for me.
Week 3: PSTs designed a WTLM prompt to give to students in their schools and reflected on students' responses	• After observing for a couple of weeks I thought I could guess how most of the students felt about math…in fact, quite a few of the students surprised me…their responses were much more positive than I expected! • When I started this activity, I seriously thought I would get nothing out of this activity, but it so happens to be quite the opposite…This activity gave me insight to what they want to do with their lives, and it made me feel that I can give them some type of knowledge relating to mathematics to help out with their career goals.
Week 6: PSTs analyzed student work on a WTLM assignment shared by a local teacher and created a grading rubric.	• Coming up with a successful rubric was stressful and a struggle for me. This is definitely something I will need to work on in my future • I think giving a writing assignment in math is definitely worth a try, but I'm just not sure how I would grade the students since I am sure to get a wide variety of answers.

Reflections generated in week 2 made it clear that answering writing prompts was a new experience for most of the PSTs. In Prompt 1, PSTs were asked to "Write a note explaining to a younger student what you need to *know* and *do* to factor $8x^2 - 2x - 21.$" They struggled to begin to write on this prompt because the activity brought to light some things they had not yet considered as teachers: how to explain something they just "knew" to others learning it for the

first time. The PSTs began to think about how the act of writing pushed them to think more deeply about what they would need to be able to do as teachers to explain concepts to a novice learner (see Table 1). From a teacher educator perspective, we were encouraged by such reflections because they reveal a growing awareness of their own expert blind spots and a consideration of ways in which they may need to rethink their own understanding of mathematics.

Responses to Prompt 2 revealed that some of the PSTs also began to reflect on the use of WTLM as a teaching tool. In this prompt, they were asked to "Explain how you know that 1/4 is bigger than 1/5." They found this task easier to write about than the first one and focused many of their reflections on the usefulness of such a task for understanding their students' thinking about the concept (rather than their own thinking). Some also discussed the place that such a writing prompt could have in the curriculum.

Prompt 3 also generated discussions of how to make expectations clear to students when giving writing tasks. The reflection for this prompt occurred as a whole class discussion that included consideration of the role that building trust between teacher and student would play when asking students to write about their mathematical knowledge. Answering this prompt alerted many PSTs to some deficits in their own mathematical knowledge. While they did not seem to feel that recognizing one's own difficulties was a bad thing, they did feel that issues could arise from asking students to write about and potentially share these vulnerable ideas.

In week 3, PSTs were asked to create their own prompt and give it to a student at their school. For this activity, many PSTs chose to create affective prompts, believing it would be easier for students to engage in these types of questions for their first experience with WTLM. They also felt that a non-content prompt could help build the trust between teacher and student in

the writing prompts discussed in week 2. The reflections revealed that some PSTs were surprised by what their students were able to write and what they were able to learn about them. Because they would soon be taking over the teaching of these students during their internship, they saw benefit in getting to know the students better using the WTLM activity. Those who were able to give the prompt to multiple students also began to consider the benefits of using writing to recognize the diversity in student thinking. Some also discussed the difficulties of motivating students to write in a mathematics class.

The last intervention activity occurred in week 6; a local teacher shared student work on a writing activity that she had used in her classroom and the PSTs were asked to create a rubric to assess the activity. This task allowed PSTs to consider potential difficulties with grading or giving feedback to an open-ended problem like a WTLM prompt. Most of their reflections on this activity focused on concerns about the time it might take to create a rubric and about their ability to create a fair rubric that could be applied to diverse student responses. For example, one student shared that, "Coming up with a successful rubric was stressful and a struggle for me," and another stated, "I'm just not sure how I would grade the students since I am sure to get a wide variety of answers." Such responses suggest that the idea of open-ended assignments and grading in mathematics was new to many of the PSTs and something with which they may need more experience.

Post Perspectives on WTLM

Following their engagement in multiple WTLM tasks and reflective discussions, we asked the PSTs to write a final reflection on the use of writing as a tool for learning mathematics. Several changes were identified in conceptions on the use and role of writing in a mathematics classroom. Before their work with WTLM, the PSTs were willing to accept writing in the

mathematics classroom for specific purposes, such as tools for explaining proofs or remembering procedures. After engaging in WTLM tasks and reflection during the methods course, however, the PSTs moved to a greater acceptance of writing in mathematics; more specifically, they saw writing as a way to build connections and assess and improve student understanding.

Although expansion of belief was the intent in this project and therefore expected, we share the details of these changes to illustrate specific ways that engagement with writing-related activities influenced PSTs' perspectives on WTLM. We present below three ways in which we have classified the conceptions of writing in mathematics held at the end of the methods course.

Writing as a tool for making connections. From an initial reluctance to see writing as a meaningful task in the mathematics classroom, the PSTs moved toward an acceptance of WTLM as a way to build connections between mathematics and other subjects. For example, one PST, who initially claimed that she did not want "to let any added writing detract from what concepts the student really needs to learn," explained her new perspective as: "Mathematics is all about communication. When we assign problems for students to solve, they need to communicate not only their answer, but also the work involved in solving it. Using writing is just a different way to achieve this goal."

The connections referred to by the PSTs were often general in focus, with an emphasis on the link between "all different types of math" and "the real world." As one PST explained,

> When students write in mathematics, it can help them to see how things they do every day involve mathematics. When they only see a formula, graph, or some expression on a sheet of paper or on the board, there is much less of an opportunity for students to see the relation to their life or to an application in the real world.

PSTs who accepted the connections offered by writing demonstrated a more holistic view of

literacy in and across the content areas. Providing students with opportunities to make those connections supports a mutually beneficial relationship between different content areas.

Several of the PSTs now felt that students, with experience, would develop a broader view of both writing and mathematics. As one explained:

> Students may be resistant at first to the idea of writing in Math, but soon will learn that it has [its] place. Just as Math has a place in all the other [subjects]. They are not mutually exclusive ideas that [are] just learned in school, but concepts and ideas that are important in every aspect of their lives.

Through experience with WLTM, PSTs moved beyond believing words belong to the English classroom and numbers to the mathematics classroom.

Writing as a tool for student assessment. The PSTs demonstrated an increased acceptance of WTLM's potential as an assessment tool. As one PST explained, through writing, students "are able to clarify their answers, and their thinking process. This is awesome…[since] we, as teachers, are not always able to see how a student is thinking or how they are coming up with the answer." Another stated that this insight was important because teachers could "see where students are in their thinking and whether or not they are understanding the concepts or simply following a procedure." As an assessment tool, teachers saw potential for recognizing students ways of thinking about mathematics.

One PST viewed WTLM tasks as particularly beneficial for working with a whole class of diverse learners, noting, "Being with so many students, it is hard for a teacher to check on every student and make sure they know the things we taught them. WTLM is a good way to check on each student." Another elaborated on this point:

> When a student gets a normal math problem wrong, the teacher knows that the student

> doesn't know how to solve that problem. If the student wrote out how to solve the problem, the teacher can see specifically why that student got that problem wrong, and where they need help. Besides helping teachers learn more about their students and their mathematical knowledge, WTLM gives students the chance to explain their thinking.

The PSTs believed that the ability to follow students' thinking was a benefit in their future teaching, supporting the use of WTLM in the mathematics classroom to assess understanding.

When viewing writing as an assessment tool, not all of the PSTs agreed that it would be easy to implement or useful in the classroom. Their primary resistance was connected to the act of grading writing tasks. Several of the PSTs noted clear drawbacks in "the extra time it takes to implement [writing] in the classroom and how hard it is to grade it and come up with a fair rubric that incorporates every possible response." They were not comfortable with qualitative grading rubrics, so they worried about the time and skill needed to create a fair and useful one for WTLM tasks. They also demonstrated a heightened concern for writing skills, and grammar was noted as an issue that the PSTs felt compelled to grade. One PST suggested that grammar was a "big topic" when using WTLM, questioning "how hard" grammar should be graded on a mathematical assignment and noting, "You don't want to scare students away with strict rules, but you want them to be writing correctly." Another added,

> I believe that the proper use of grammar is crucial in whatever you do, but again, we are not language teachers. If there 1 or 2 minor grammar issues, I think it would be OK to just correct the problem, but I personally think that a few points can be taken off if the students excessively do not use proper grammar.

Many of these comments were motivated by the in-class exercise creating rubrics for actual writing samples where grammatical mistakes were prevalent and distracting for the PSTs.

Writing as a tool for improving student learning. At the end of the methods course, the PSTs continued to see the potential benefits of writing to student learning, but now focused on writing as an integral component of mathematics. The PSTs expressed a desire for WTLM to be incorporated "as a normal part of our classroom instead of something that is thrown in at random times." Some reflections suggested that writing was not only a sufficient but necessary tool for pushing students to move beyond rote learning in mathematics. One teacher's final comments summarized this belief powerfully:

> Provide students with a reason to think about Math in a deeper context and give them the tools to do it and they will; don't provide them a reason and they will do just the bare minimum to pass a course, pass a test, and quickly forget all their knowledge.

Several PSTs reflected on the need to make certain that writing increased conceptual understanding and be used consistently in the curriculum.

If used for the correct purposes in the classroom, the PSTs expressed confidence that students would benefit greatly from engaging in writing tasks. For example, one PST, who admitted struggling in high school to understand why her teacher included writing in a mathematics class, seemed certain that she should use writing in her own classroom and that she could help her students see its benefits. She shared, "Writing provides [students] with a chance to expand on their critical thinking skills and problem solving…Hopefully, students will learn that writing in math will help them understand the concepts at a deeper level."

Others focused on the opportunities that WTLM gives students to explain their thinking. For example, one PST posited that the act of writing ideas down in words "could help clarify what the student is thinking not only to someone else, but to the person who is doing the writing," while another teacher explained, "Writing provides [students] with a chance to expand

on their critical thinking skills and problem solving." One PST, who initially reported that writing was too time-consuming to use, shared the following in week six:

> I believe that WTLM is an exceptional tool to use for your students. I believe that students can use writing to learn mathematics in a different light because they are reflecting on what they have learned and they are intently thinking about the mathematical concepts. Using writing can allow students to focus on mathematics, and that leads to greater understanding.

Seeing mathematics from a different perspective through their own engagement in and reflection on WTLM tasks allowed the PSTs to consider writing from a different perspective, as well, supporting that same goal in their future work with students.

Summary

As part of our larger research goals, we hypothesized that incorporating and eventually normalizing the use of writing in mathematics offers multiple benefits to PSTs as a tool for understanding the diverse ways in which students learn and the need for addressing diverse needs through one's teaching practices. However, before asking teachers to incorporate a new tool into the classroom, it is important to first develop their experiences with the tool and to then examine their conceptions of its usefulness. Engagement with and reflection on WTLM over several weeks allowed PSTs to reconsider their understanding of the place of writing in the mathematics classroom. Through experiences, discussions and reflections, the PSTs were able to view WTLM as a potentially meaningful addition to their future classroom. Although changes in PSTs' beliefs about WTLM was the intent of this project and therefore expected, we feel it is useful for teacher educators to examine specific ways in which even a small amount of engagement with writing-related activities influenced PSTs' perspectives on the use of a new (to them) tool in teaching

and learning. Despite initial reluctance and uncertainty, almost all of the PSTs were able to identify value in writing following their own experiences with WTLM over a six-week methods course.

PSTs were willing to accept writing as a tool in the mathematics classroom if they believed it was implemented effectively and purposefully. Part of purposeful implementation was the use of WTLM as a "normal" and continuous component of the classroom curriculum. Several concerns about WTLM persisted over the course, specifically related to implementation, time, and grading, but these concerns did not appear to create an aversion to incorporating WTLM tasks in their teaching. Many concerns were closely linked to PSTs' past experiences, suggesting that additional engagement with WTLM and experiences implementing WTLM tasks are needed to expand PSTs' views on writing as a learning and assessment tool in mathematics.

As teacher educators, we know that in order to disrupt the basic schema of "knowing" mathematics, PSTs need opportunities to engage with mathematics in ways that interrogate and reframe their current understandings. We have begun to explore ways in which we can perturb PSTs' ways of thinking about both teaching and learning mathematics by engaging them in WTLM tasks that focus on their own understanding of mathematical concepts and procedures and by having them read and reflect on students' responses to similar tasks. Although not explored fully within the scope of this article, we have referenced ways in which the PSTs' reflections on their own experiences with WTLM served to reframe elements of their pedagogical and content knowledge. This was evidenced by reflections where PSTs began to consider the pedagogical decisions they would make to develop students' thinking. Asking the PSTs to reflect on their own experiences with these tasks allowed them to consider things like the complexity of the tasks, students' thinking strategies, and strategies for teaching

mathematical concepts, all of which are important components of pedagogical content knowledge (Chick, Baker, Pham, & Cheung, 2006). Further research is needed, however, to examine in more detail how using writing tasks in teacher preparation programs can support an expanded view of teaching and learning that, in turn, may enhance PSTs' pedagogical content knowledge and help address the issue of the expert blind spot.

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