Math 462 Fall 11
Sample Midterm Questions

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(1) Give the definition of a regular parameterized differentiable curve.

(2) Give the definition of the arc length parameter.

(3) Can every curve be reparameteried by arc length?

(4) Give an equation for the tangent line of a regular curve $\alpha : I \to \mathbb{R}^3$ at the point $t \in I$.

(5) What can you say about a curve α whose second derivative $\alpha'' = 0$.

(6) Give a parameterization of a circle of radius a clockwise/counterclockwise

(7) Give a parameterization of a helix of radius a with height c.

(8) Let α be a curve parameterized by arc length. Define the tangent, normal and bi-normal vectors $\mathbf{t}, \mathbf{n}$ and $\mathbf{b}$ of α at $\alpha(t)$. Do you need any additional assumptions for this? What would happen if the curve where not parametrized by arc length?

(9) Define curvature and torsion.

(10) What is the torsion of a planar curve?

(11) Write down the Frenet formulas.

(12) State the fundamental theorem of the local theory of curves.

(13) Give a curve which has constant curvature and torsion.

(14) Is there a curve $\alpha : [-1, 1] \to \mathbb{R}^3$ whose curvature is $\sin(t)$ and torsion $\cos(t)$, how about curvature t^2 and torsion t^3.

(15) True of false. To first order a curve lies in a plane. To third order a curve lies in a half space. To second order the curve lies in the normal plane. The osculating plane is defined to be the plane containing the tangent and the normal vector.

(16) Define a regular surface

(17) Give examples of regular surfaces.

(18) Give coordinate charts which cover S^2.

(19) Give the definition of a rotational surface. Is a rotational surface regular? Give coordinate patches which cover a given a rotational surface.

(20) True or false. Every regular surface has a global parameterization. One of the projections of a regular surface to the three coordinate planes is a graph. Locally a surface is a graph. The inverse image of a regular value is a regular surface. The inverse image of a critical value is necessarily not regular.

(21) Give the definition of the tangent plane T_pS to a point p of a regular surface S.

(22) Let $\phi : S_1 \to S_2$ be a differentiable map between two regular surfaces. How is the differential $d\phi(p) : T_pS_1 \to T_{\phi(p)}S_2$ defined.

(23) When is a regular surface orientable? Give examples of orientable and non-orientable surfaces. Explain your examples.

(24) Give the definition of the first fundamental form.

(25) Calculate the first fundamental form of the upper sheet of the hyperboloid $x^2 + y^2 - z^2 = 1$.

(26) What is the Gauss map? Give the Gauss map for the cylinder, a graph and the sphere.

(27) Give the definition of the second fundamental form.

(28) Give a geometric interpretation of $II_p(v)$.

(29) Derive the Euler formula. Let e_1, e_2 be a basis of $T_p(S)$ such that e_1 and e_2 are principal curvature directions of curvature k_1, k_2. Calculate the normal curvature of the curve α through $p = \alpha(0)$ if $\alpha'(0) = cos(\theta)e_1 + \sin(\theta)e_2$.

(30) Give the definition of Gauss curvature and mean curvature.

(31) Calculate the Gauss and mean curvature of the sphere and the cylinder of radius one.