Math 572
Spring 20
Take-home final
Faculty: R. Kaufmann

Name:

Signature:

Student ID Number:

Directions: Please send a pdf (scan) file to rkaufman@math.purdue.edu by Tuesday 05/05 by 5:30 pm.

Use this page as a cover page.

Problem 1: a) Give the axioms of a homology theory using functorial formulation.
b) How are they related and different from those of a cohomology theory.

Problem 2: Are the chain groups of singular and simplicial and their relative version chains free? If so give a basis.

Problem 3: a) Prove that the singular homology groups are functorial.
b) How is the natural transformation of the axioms defined?

Problem 4: Give a sketch of the proof that for a triangulable space the singular and simplicial homology are isomorphic.

Problem 5: What is the technical role of $\mathcal{A}$–small simplices in singular homology. What property of simplices is being used?

Problem 6: a) How is the chain complex of a CW complex defined. Give the chain groups and a definition of the differential. Show that the differential squares to 0.
b) Why the chain groups free? Give a basis and an interpretation of the differential on the basis elements.

Problem 7: Give an overview on how to obtain the Mayer-Vietoris sequence.

Problem 8: Describe the steps in the Künneth formulas for the homology and co–homology of topological spaces. In particular, what are the roles of the Alexander-Whitney maps and the Eilenberg-Zilber maps?

Problem 9: Calculate the homology and cohomology of the torus, the Klein bottle and $\mathbb{RP}^2$ with $\mathbb{Z}/2\mathbb{Z}$ coefficients using the univesal coefficient theorems.

Problem 10: Compute the homology and the cohomology ring of $T^n = (S^1)^{\times n}$.