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Math 598, Fall 2018

Problem Set 3

Problems

Problem 1: Read the details of the isomorphism between homology groups.

Problem 2: Compute the homology of the simplicial complex that is a square.

Problem 3: Let K be a chain complex with finitely many p–simplices in each degree, so that the chain groups $C_p(K)$ have finite rank. Define the Euler characteristic of K to be

$$\chi(K) = \sum_i (-1)^i b_i$$

where b_i is the Betti number of $H_i(K)$.

Using the fact that *if G is torsion free* $H_i(K;G) \simeq H_i(K) \otimes G$ show that for any field k of characteristic 0

(1)
$$b_i = dim_k(H_i(K;k))$$

(2)
$$\chi(K) = \sum_i (-1)^i dim_k(C_i(K;k))$$

Problem 4: Prove that $\check{C}ech(r) \subseteq VR(r) \subseteq \check{C}ech(\sqrt{2}r)$ $VR(r)$ is the Vietoris-Rips and $\check{C}ech(r)$ is the Čech complex.

Problem 5: How are the above two complexes related to the clique complex?