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**Math 598, Fall 2018**

# Problem Set 1

## Problems

Problem 1: Show that the formula

$$\partial[v_0, \ldots, v_n] = \sum_{i=0}^{n} (-1)^i [v_0, \ldots, v_{i-1}, \widehat{v_i}, v_{i+1}, \ldots, v_n]$$

is well defined on oriented simplices. First state what the problem is.

Problem 2: Show that $\partial^2 = 0$.

Problem 3: Give the matrix $M$ representing $\partial$ for your choice of basis of $\bigoplus_i C_i(\mathscr{S}_{\Delta^2})$ and show that indeed $\partial^2 = 0$ by computing $M^2$.

Problem 4: Compute the homology of a square with one diagonal.

Problem 5: In a simplicial graph, show that contracting a contractible edge on a graph does not change its homology groups.

(Note: a simplicial graph is a simplicial complex with only 0 and 1 simplices. A 1 simplex, aka. edge, is contractible if the vertex map identifying the two vertices of the one simple is bijection on the set of all other 1-simplices.)

Problem 6: For a simplicial complex $\mathscr{S}$ show that $H_p(\mathscr{S})$ only depends on the $p + 1$ skeleton.