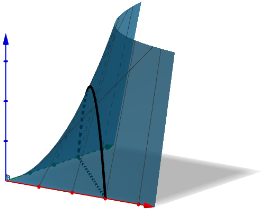
The Setting/History: $G = SL(n, \mathbb{R})$

- upper triangular, 1's on diagonal
- totally nonnegative part: all minors ≥ 0
- $W = S_n$, generators $s_1 = (1\ 2)$, $s_2 = (2\ 3)$,...
- Conjecture (Fomin Shapiro): regular CW complex (proved by Hersh)

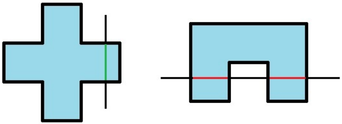


The Current Goal:

$$f_{(1,2,1)} = \begin{bmatrix} 1 & t_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & t_3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & t_1 + t_3 & t_1 t_2 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Davis, Hersh, Miller

- Conjecture: fibers of $f_{(i_1,\ldots,i_d)}$ stratify into regular cells
- Strata correspond to subwords Q of $(i_1, \ldots, i_d)$



The Method:

Monotone sets (Basu, Gabrielov, Vorobjov)

X monotone $\Rightarrow$ X a regular cell

The Results thus far:

Examples that work

- $Q = (1,3,1,2,3,2)$: $\{(t_1, \frac{a_3(a_4-t_6)}{a_2+(a_4-t_6)}, a_1 - t_1, a_2 + (a_4 - t_6), \frac{a_2 a_3}{a_2+(a_4-t_6)}, t_6) \mid 0 < t_1 < a_1, 0 < t_6 < a_4\}$
- $Q = (1,2,1,2,1)$: $\{(a_1 - \frac{(a_3-t_5)t_4}{(a_2-t_4)}, a_2 - t_4, \frac{a_2(a_3-t_5)}{a_2-t_4}, t_4, t_5) \mid 0 < t_5 < a_3, 0 < t_4 < \frac{a_1 a_2}{a_1+(a_3-t_5)}\}$

An example that doesn't

- $Q = (2,3,1,2,3,1,2)$: $\{(\frac{a_2 a_5(a_6-t_7)}{a_1 a_4+(a_1+a_5)(a_6-t_7)}, a_1 + \frac{a_5(a_6-t_7)}{a_4+(a_6-t_7)}, a_3\frac{a_1 a_4+(a_1+a_5)(a_6-t_7)}{a_1(a_2+a_4)+(a_1+a_5)(a_6+t_7)}, (a_4 + (a_6 - t_7))(1 + \frac{a_1 a_2}{a_1 a_4+(a_1+a_5)(a_6-a_7)}), \frac{a_4 a_5}{a_4+(a_6-t_7)}, \frac{a_1 a_2 a_3}{a_1(a_2+a_4)+(a_1+a_5)(a_6-t_7)}, t_7) \mid 0 < t_7 < a_6\}$