Sequence Mini-Exercises

MA 504 Problem Session

September 29, 2022

1. Is the following an acceptable corollary to Rudin Theorem 3.14?

   Corollary. *Let $\{s_n\}$ be a sequence of real numbers.*

   (a) Suppose $\{s_n\}$ is monotone increasing. Then $\{s_n\}$ converges if and only if the sequence is bounded above.

   (b) Suppose $\{s_n\}$ is monotone decreasing. Then $\{s_n\}$ converges if and only if the sequence is bounded below.

2. Let $\{s_n\}$ be a sequence of real numbers. Say that $\{s_n\}$ is monotone increasing and that $\lim_{n\to\infty} s_n = L$. Prove that for all n, $s_n \leq L$.

3. (Rudin p. 78 #1) Prove that the convergence of $\{s_n\}$ implies the convergence of $\{|s_n|\}$. Is the converse true?

4. Give five examples (distinct in spirit) of sequences of real numbers whose sets of subsequential limits E^* are such that $1 < E^* < \infty$.

5. Craft a sequence $\{s_n\}$ for which both of the following hold:

   (i) $\limsup_{n\to\infty} s_n \neq \liminf_{n\to\infty} s_n$

   (ii) Neither $\limsup_{n\to\infty} s_n$ nor $\liminf_{n\to\infty} s_n$ are in the range of $\{s_n\}$.

6. Let X be a subset of a metric space, and let $p \in \overline{X}$. Show that there exists a sequence of points of X converging to p.