Compactness of the set of iso-resonant potentials for Schrödinger operators in low dimensions

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In joint work with R. Wolf, we prove compactness of a restricted set of real-valued, compactly supported potentials $V$ for which the corresponding Schrödinger operators $H_V$ have the same resonances, including multiplicities. More specifically, let $B_R(0)$ be the ball of radius $R > 0$ about the origin in $\mathbb{R}^d$, for $d = 1$ or $d = 3$. For any real-valued potential $V_0 \in C_0^\infty(B_R(0))$, let $I_R(V_0)$ be the set of real-valued potentials in $C_0^\infty(B_R(0))$ so that the corresponding Schrödinger operators have the same resonances, including multiplicities, as $H_{V_0}$. We prove that the iso-resonant set $I_R(V_0)$ is a compact subset of $C_0^\infty(B_R(0))$ in the $C^\infty$-topology. Extensions to sets of less regular potentials in various Sobolev spaces are also obtained.