Spectrally unstable domains

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Let $H$ be a separable Hilbert space, $A_c : \mathcal{D}_c \subset H \to H$ a densely defined unbounded operator, bounded from below, let $\mathcal{D}_{\min}$ be the domain of the closure of $A_c$ and $\mathcal{D}_{\max}$ that of the adjoint. Assume that $\mathcal{D}_{\max}$ with the graph norm is compactly contained in $H$ and that $\mathcal{D}_{\min}$ has finite positive codimension in $\mathcal{D}_{\max}$. Then the set of domains of selfadjoint extensions of $A_c$ has the structure of a finite-dimensional manifold $\mathfrak{SA}$ and the spectrum of each of its selfadjoint extensions is bounded from below. If $\zeta$ is strictly below the spectrum of $A$ with a given domain $\mathcal{D}_0 \in \mathfrak{SA}$, then $\zeta$ is not in the spectrum of $A$ with domain $\mathcal{D} \in \mathfrak{SA}$ near $\mathcal{D}_0$. But $\mathfrak{SA}$ contains elements $\mathcal{D}_0$ with the property that for every neighborhood $U$ of $\mathcal{D}_0$ and every $\zeta \in \mathbb{R}$ there is $\mathcal{D} \in U$ such that $\operatorname{spec}(A_{\mathcal{D}}) \cap (-\infty, \zeta) \neq \emptyset$. We characterize these "spectrally unstable" domains as being those satisfying a nontrivial relation with the domain of the Friedrichs extension of $A_c$.

This is a situation that arises in the context of elliptic semibounded cone operators on compact manifolds with conical singularities.