15. Let $\boxed{A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}}$. Then A^6 is:

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

A. $\begin{bmatrix} 1 & 1 \\ 0 & 64 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -63 \\ 0 & 64 \end{bmatrix}$

$$D^2 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix}$$

C. $\begin{bmatrix} -1 & -63 \\ 0 & -64 \end{bmatrix}$

$$D^3 = D \cdot D^2 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix} = \begin{bmatrix} \lambda_1^3 & 0 \\ 0 & \lambda_2^3 \end{bmatrix}$$

D. $\begin{bmatrix} 1 & 63 \\ 0 & 64 \end{bmatrix}$

E. $\begin{bmatrix} -1 & 63 \\ 0 & -64 \end{bmatrix}$

$$\underset{P}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}} \underset{D^6}{\begin{bmatrix} 1 & 0 \\ 0 & 64 \end{bmatrix}} \underset{P^{-1}}{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}$$

$$D^k = \begin{bmatrix} \lambda_1^k & 0 \\ 0 & \lambda_2^k \end{bmatrix}$$

16. Which of the following matrices are diagonalizable?

(i) $\begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 6 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 2 \end{bmatrix}$

A. (i) and (iii) only

B. (iii) and (iv) only

C. (ii) and (iii) only

D. (i), (ii) and (iv) only

E. (i) , (ii), (iii) and (iv)

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$\lambda_1 = 1$

$(A - \lambda_1 I)\vec{x} = 0$

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \qquad 0x_1 + 1x_2 = 0$$

e. vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\uparrow$

$\lambda_2 = 2$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \qquad -x_1 + x_2 = 0$$

e. vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\uparrow$

$$A = P D P^{-1}$$

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$\uparrow \quad \uparrow$

$$P^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad D^6 = \begin{bmatrix} 1 & 0 \\ 0 & 64 \end{bmatrix}$$

$$A = P D P^{-1}$$

$$A^6 = (\boxed{P} D \cancel{P^{-1}}) \cdot (\cancel{P} D \cancel{P^{-1}}) \cdots (\cancel{P} D \boxed{P^{-1}}) = P D^6 P^{-1}$$