$AA = A[\vec{a}_1 \cdots \vec{a}_n] = [A\vec{a}_1 \cdots A\vec{a}_n] = 0$

1. Let A be a 4×4 matrix. Which of the following statements is always TRUE?

A. The reduced echelon form of A has at least 1 pivot. $\rightarrow$ $A = 0$

B. If $\boxed{A^2 = 0}$ then the system $\boxed{A\mathbf{x} = 0}$ has only the trivial solution.

C. If $A^2 = A$ then the reduced echelon form of A is I_4. $\rightarrow$ $A = 0$

$(\det A) = \pm 1$

D. If $A^2 = I_4$ then the reduced echelon form of A is I_4.

E. A is diagonalizable over the complex numbers.

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

2. For which of the following five values of the parameter a is the set $\left\{ \begin{bmatrix} a \\ a \\ a \end{bmatrix}, \begin{bmatrix} 1 \\ a \\ a \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ a \end{bmatrix} \right\}$ linearly independent?

(i) $a = 0$
(ii) $a = 1$
(iii) $a = 2$
(iv) $a = 3$
(v) $a = 4$

A. (iv) and (v) only

B. (iii) and (v) only

C. (i), (ii), and (iv) only

D. (ii), (iii), and (iv) only

E. (i), (ii), (iii), and (iv) only

$(m \times n)(n \times 1) \to m \times 1$

3. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and $T(\mathbf{x}) = A\mathbf{x}$, where A is the standard matrix for T. Which of the following statements must be TRUE?

(i) A is an $n \times m$ matrix.

(ii) If the columns of A are linearly independent, then T is one-to-one.

(iii) If $m > n$, then T is onto.

(iv) If $n > m$, then T is one-to-one.

(v) If the columns of A span $\mathbb{R}^m$, then T is onto.

$T(x_1) = T(x_2)$

$\Rightarrow x_1 = x_2$

Col A = Range(T)

A. (i), (ii), (iii), (iv), and (v)

B. (i), (ii), and (iii) only

C. (ii) and (v) only

D. (ii), (iii), and (iv) only

E. (iii) and (iv) only

4. Consider the system of linear equations

$$\begin{aligned} x + 3y - z &= 5 \\ 2x + 5y + az &= 9 \\ x + y + a^2 z &= a \end{aligned}$$

Under which conditions does this system have infinitely many solutions?

A. $a \neq -1$

B. $a \neq 3$

C. $a = -1$

D. $a = 3$

E. $a \neq -1$ and $a \neq 3$

9. Which of the following sets of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a subspace of $\mathbb{R}^3$?

deg 2.

(i) The set of all vectors satisfying $\boxed{x + y + z = 0.}$

LINEAR RELATION

(ii) The set of all vectors satisfying $xy - z = 0$.

$\boxed{x}\boxed{y} = z$

(iii) The set of all vectors satisfying $xyz = 0$.

(iv) The set of all vectors satisfying $x + y - z^2 = 0$.

(v) The set of all vectors satisfying $x + y - z \geq 0$.

$(1,1,0) - (3,0,0) = (-2,1,0)$

$-2+1-0 < 0$

A. (i) only

B. (i), (iii), (iv), and (v) only

C. (i) and (ii) only

D. (i) and (v) only

E. None of them is a subspace of $\mathbb{R}^3$

$(1,1,1) + (1,2,2) = (2,3,3)$

10. The points $(1, 2)$, $(2, 4)$, $(4, 5)$, and $(5, 7)$ are the vertices of a parallelogram on the coordinate plane. What is the area of this parallelogram?

A. -3

B. 3

C. 6

D. 0

E. 1

15. Let the transformation $T : \mathbf{x} \to A\mathbf{x}$ be the composition of a rotation by angle $\theta \in (-\pi, \pi]$, followed by a scaling by the factor $r > 0$. If $A = \begin{bmatrix} -9\sqrt{3} & 9 \\ -9 & -9\sqrt{3} \end{bmatrix}$, find θ and r.

$$\det(A - \lambda I) = (-\lambda - 9\sqrt{3})^2 + 9^2 = 0$$

A. $\theta = -5\pi/6, \quad r = 18$

~~B. $\theta = \pi/3, \quad r = 9$~~

C. $\theta = 5\pi/6, \quad r = 18$

D. $\theta = \pi/6, \quad r = 18$

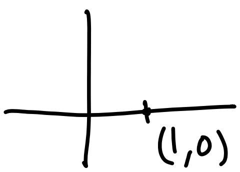
E. $\theta = -\pi/6, \quad r = 18$

$$(\lambda + 9\sqrt{3})^2 = -9^2$$

$$\Rightarrow \lambda = -9\sqrt{3} \pm 9i$$

(1, 0)

$$\sqrt{(9\sqrt{3})^2 + 9^2} = \sqrt{2^2 \cdot 9^2} = 18$$



16. Consider the following system of differential equations

$$x'(t) = 4x(t) + 2y(t)$$
$$y'(t) = 2x(t) + 4y(t)$$

with the initial condition $x(0) = 8, y(0) = 2$. What is the value of $x(1) + y(1)$?

A. $10e^6$

B. $10e^{-6}$

C. $6e^6$

D. $10e^6 + 6e^2$

E. $6e^6 + 10e^2$

$$A\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-9\sqrt{3} & 9\\ -9 & 9\sqrt{3}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-9\sqrt{3}\\ -9\end{bmatrix}$$