Homework 1

MA 30300 (Fall 2025, §764)

August 29th, 2025 (Updated: Sept 5th)

Instructions

- Due: Saturday, September 6th at 11 PM Eastern Time.
- Total Score: 20 points.
- Section numbers and problem numbers below are as in *Differential Equations and Boundary Value Problems* (6th Edition) by C. Henry Edwards, David E. Penney, and David Calvis.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem §5.1 #25. [5 points] Verify that $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions to the system

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix} \mathbf{x}, \tag{1}$$

where

$$\mathbf{x}_1(t) = \begin{bmatrix} 3e^{2t} \\ 2e^{2t} \end{bmatrix}, \qquad \mathbf{x}_2(t) = \begin{bmatrix} e^{-5t} \\ 3e^{-5t}. \end{bmatrix}$$

Then, use the Wronskian (see §5.1 of the book) to show that they are linearly independent. Finally write the general solution to the system (1).

Problem §5.2 #2. [5 points] Find the general solution of the following system:

$$\begin{aligned} x_1' &= 2x_1 + 3x_2, \\ x_2' &= 2x_1 + x_2. \end{aligned}$$

Draw the typical solution curves for this system. You may use a graphing calculator or computer system if you prefer, but a hand-drawn rough diagram is sufficient.

Problem §5.2 #12. [5 points] Find the general solution of the following system:

$$\begin{aligned} x_1' &= x_1 - 5x_2, \\ x_2' &= x_1 + 3x_2. \end{aligned}$$

Draw the typical solution curves for this system. You may use a graphing calculator or computer system if you prefer, but a hand-drawn rough diagram is sufficient.

Problem §5.2 #17. [5 points] Find the general solution of the following system:

$$\begin{aligned} x_1' &= 4x_1 + x_2 + 4x_3, \\ x_2' &= x_1 + 7x_2 + x_3, \\ x_3' &= 4x_1 + x_2 + 4x_3. \end{aligned}$$