

Written Homework 11 (H11)

MA 30300 (Fall 2025, §764)

November 19th, 2025 (Updated: Nov 20th)

Instructions

- Due: Saturday, December 6th at 11 PM Eastern Time.
- Total Score: 40 points.
- Section numbers and problem numbers below are as in *Differential Equations and Boundary Value Problems* (6th Edition) by C. Henry Edwards, David E. Penney, and David Calvis. There may be typos below; in this case, always default to the numbers/signs/wording from the textbook.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in L^AT_EX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem §9.7 #2. [10 points] Solve the following Dirichlet problem:

$$\begin{aligned}u_{xx} + u_{yy} &= 0 & (0 < x < a \text{ and } 0 < y < b), \\u(x, 0) &= u(x, b) = u(a, y) = 0, \\u(0, y) &= g(y).\end{aligned}$$

Problem §9.7 #8. [10 points] Find a solution to Laplace's equation $u_{xx} + u_{yy} = 0$ in the semi-infinite strip $0 < x < a, y > 0$ that satisfies $u(x, y)$ is bounded as $y \rightarrow \infty$ as well as the boundary conditions

$$u_x(0, y) = u_x(a, y) = 0, \quad u(x, 0) = f(x).$$

Problem §10.1 #2. [10 points] Verify that the eigenvalues and eigenfunction for following Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad y(0) = y'(L) = 0,$$

are

$$\lambda_n = \frac{(2n-1)^2\pi^2}{4L^2}, \quad y_n(x) = \sin \frac{(2n-1)\pi x}{2L},$$

for $n \geq 1$.

Problem §10.1 #8. [10 points] Represent the constant function $f(x) = 1$ as a series of eigenfunctions of the Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad y'(0) = hy(L) + y'(L) = 0 \quad (h > 0),$$

which has eigenvalues and eigenfunctions

$$\lambda_n = \frac{\beta_n^2}{L^2}, \quad y_n(x) = \cos \frac{\beta_n x}{L},$$

for $n \geq 1$ where β_n is the n th positive root of $\tan x = hL/x$.