# Written Homework 3 (H3)

MA 30300 (Fall 2025, §764)

September 14th, 2025

## Instructions

- Due: Saturday, September 20th at 11 PM Eastern Time.
- Total Score: 40 points.
- Section numbers and problem numbers below are as in *Differential Equations and Boundary Value Problems* (6th Edition) by C. Henry Edwards, David E. Penney, and David Calvis. There may be typos below; in this case, always default to the numbers/signs/wording from the textbook.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

**Problem 0.** [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

**Problem §6.1 #7.** [10 points] Find the critical points of the following autonomous system:

$$\frac{dx}{dt} = x - 2y, \qquad \frac{dy}{dt} = 4x - x^3.$$

Using this information, identify the phase portrait of this system from among the figures on p. 383 of the textbook.

**Problem §6.1 #20.** [10 points] Consider the following autonomous system:

$$\frac{dx}{dt} = y, \qquad \frac{dy}{dt} = -5x - 4y.$$

Determine whether the critical point $(0, 0)$ is stable, asymptotically stable, or unstable as well as whether it is a node, a saddle point, a spiral point, or a center. Draw a phase diagram and a direction field (you may use a computer system or graphing calculator if you like, but a suitable hand-drawn diagram is sufficient).

**Problem §6.2 #16.** [10 points] Consider the following autonomous system:

$$\frac{dx}{dt} = x - 2y + 1, \qquad \frac{dy}{dt} = x + 3y - 9.$$

Determine the critical point and use the Theorem 2 from the book (which characterizes the stability of almost linear systems) to determine what you can about the stability and type of the critical point (as in the previous question).

**Problem §6.2 #20.** [10 points] Consider the following autonomous system:

$$\frac{dx}{dt} = 6x - 5y + x^2, \qquad \frac{dy}{dt} = 2x - y + y^2.$$

Investigate the behaviour of the critical point $(0, 0)$ using Theorem 2. Use a computer system[1] or a graphing calculator to draw the phase portrait and verify your conclusions[2].

[1] For example, the one at https://aeb019.hosted.uark.edu/pplane.html.

[2] You do not need to submit drawings made by a computer system/graphing calculator