Written Homework 4 (H4)

MA 30300 (Fall 2025, §764)

September 23rd, 2025

Instructions

- Due: Tuesday, September 30th (**note:** unusual day!) at 11 PM Eastern Time.
- Total Score: 40 points.
- Section numbers and problem numbers below are as in *Differential Equations and Boundary Value Problems* (6th Edition) by C. Henry Edwards, David E. Penney, and David Calvis. There may be typos below; in this case, always default to the numbers/signs/wording from the textbook.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem §6.3 #17. [10 points] Consider the following autonomous system:

$$\frac{dx}{dt} = x^2 - 2x - xy, \qquad \frac{dy}{dt} = y^2 - 4y + xy.$$

This is an alternative predator-prey model where $y(t)$ represents the number of predators and $x(t)$ represents the number of prey. Show that $(3, 1)$ is a critical point, and that its linearization at $(3, 1)$ is

$$\frac{du}{dt} = 3u - 3v, \qquad \frac{dv}{dt} = u + v.$$

Show that the eigenvalues of the linearization are $2 \pm i\sqrt{2}$ and thus determine $(3, 1)$ is a spiral source for the above systems.

Problem §6.3 #26. [10 points] Consider the following two-population system:

$$\frac{dx}{dt} = 2x - xy, \qquad \frac{dy}{dt} = 3y - xy.$$

Determine the nature of x or y in the absence of the other (exponential or logistic) and the nature of their interaction (competition, cooperation and, predation). Then, find and characterize the system's critical points (type and stability). Determine the value of x and y in which the populations can co-exist. Finally, construct a phase plane portrait (you may use a computer system or graphing calculator if you wish).

Problem §7.1 #18. [10 points] Determine the Laplace transform of the following function.

$$f(t) = \sin 3t \cos 3t.$$

To do so, you may use the Laplace transform table (Figure 7.1.2 of the book) if necessary without further justification.

Problem §7.1 #32. [10 points] Determine the inverse Laplace transform of the following function.

$$F(s) = 2s^{-1}e^{-3s}.$$

To do so, you may use the Laplace transform table (Figure 7.1.2 of the book) if necessary without further justification.