Written Homework 7 (H7)

MA 30300 (Fall 2025, §764)

October 19th, 2025

Instructions

- Due: Saturday, October 25th at 11 PM Eastern Time.
- Total Score: 40 points.
- Section numbers and problem numbers below are as in *Differential Equations and Boundary Value Problems* (6th Edition) by C. Henry Edwards, David E. Penney, and David Calvis. There may be typos below; in this case, always default to the numbers/signs/wording from the textbook.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem §7.6 #8. [10 points] Solve the following initial value problem:

$$x'' + 2x' + x = \delta(t) - \delta(t-2), \qquad x(0) = x'(0) = 2.$$

Problem §7.6 #12. [10 points] Using Duhamel's principle to write an integral formula involving an unknown forcing function $f(t)$ for the solution of the following initial value problem:

$$x'' + 4x' + 8x = f(t), \qquad x(0) = x'(0) = 0.$$

Problem §2.5 #28, §2.6 #28. [20 points]
Consider the following IVP:

$$y' = x + \frac{1}{2}y^2, \qquad y(-2) = 0.$$

Compute an approximation for $y(2)$ using the following methods:

(a) Using the Euler method with $h = 2$.

(b) Using the improved Euler method starting with the step-size $h = 0.1$ and then successively reducing the step-size until successive approximations agree rounded off to four decimal places.

(c) Using the fourth-order Runge-Kutta method starting with the step-size $h = 1$ and then successively reducing the step-size until successive approximations agree rounded off to five decimal places.

Part (a) must be done by hand. For Part (b) and (c), you may use whichever you prefer among a scientific calculator, a computer algebra system (Mathematica, WolframAlpha etc.), or a computer program (written in, e.g., C, MATLAB, Python etc.). For full credit in (b) and (c), you need to report what system you used, an illustrative input (command, program etc.) you gave it, and record in a table the values of step-sizes that you used together with the final value of the approximation for $y(2)$ that step-size yielded. It might be helpful to read pp. 125–127 and pp. 135–137 of the book (i.e., the "Application" subsections of §§2.5–2.6).