Question Bank for Final Exam

MA 30300 (Fall 2025, §764)

December 12th, 2025 (Updated: Dec 16th)

Notes

- There will be three types of questions on the final exam: T/F (True or False), MCQs (Multiple Choice Questions), and Long Answer Questions (LAQs).
- The structure of the final exam will be as follows:
  - 10 T/Fs (1.5 points each = 15 points total).
  - 4 MCQs (5 points each = 20 points total). Each MCQ will have 5 options, of which only one is correct.
  - 3-4 LAQs (5-10 points each depending on complexity = 25 points total)
- The final exam will be worth 60 points total.
- This is a sample question bank for LAQs.
- This bank focuses on material after Midterm 2. LAQs similar to the ones that showed up in Midterm 1 and Midterm 2 may also show up in the final; for these also, please see the archives.

Long Answer

Problem 1. Using the method of separation of variables, solve the following boundary value problem for the heat equation for $u(x,t)$,

$$u_t = 100u_{xx}$$

for a rod of length $L = 3$ with insulated end-points satisfying the initial value conditions

$$u(x,0) = x^2.$$

Problem 2. Find the general solution $u(x,t)$ of the boundary value problem for the following heat equation on a rod of length L:

$$u_t = ku_{xx} \qquad (0 < x < L),$$

$$u(0,t) = u(L,t) = 0,$$

$$u(x,0) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}.$$

Problem 3. Find the solution to the wave equation $y(x,t)$ which satisfies the following boundary value problem:

$$y_{tt} = 9y_{xx} \qquad (0 < x < 1, t > 0),$$

$$y(0,t) = y(1,t) = 0,$$

$$y(x,0) = \sin \pi x - 2 \sin 3\pi x, \qquad y_t(x,0) = 12\pi \sin 2\pi x.$$

Problem 4. Obtain $u(x,y)$ that solves the Dirichlet problem for the rectangle $0 < x < 2, 0 < y < 4$ for Laplace's equation

$$u_{xx} + u_{yy} = 0,$$

with the homogeneous constraints

$$u(x,0) = u(x,4) = 0 \qquad (0 < x < 2),$$
$$u(0,y) = 0 \qquad (0 < y < 4),$$

and the non-homogeneous constraint

$$u(4,y) = y(4-y) \qquad (0 < y < 4).$$

Problem 5. Solve the Dirichlet problem for the half-infinite strip $\{x > 0,\ 0 < y < 3\}$ for Laplace's equation

$$u_{xx} + u_{yy} = 0,$$

with the constraints

$$u(x,0) = u(x,3) = 0 \qquad (x > 0),$$
$$u(0,y) = 1 \qquad (0 < y < 3),$$
$$u(x,y) \text{ is bounded as } x \to \infty.$$

Problem 6. Solve the Dirichlet problem for the half-infinite strip $\{x > 0,\ 0 < y < 3\}$ for Laplace's equation

$$u_{xx} + u_{yy} = 0,$$

with the constraints

$$u_y(x,0) = u_y(x,3) = 0 \qquad (x > 0),$$
$$u(0,y) = 1 \qquad (0 < y < 3),$$
$$u(x,y) \text{ is bounded as } x \to \infty.$$

Note: this is not the same as the previous problem!

Problem 7. Solve the following Sturm-Liouville problem:

$$y'' + \lambda y = 0,$$

$$y'(0) = 0, \qquad y(10) + 5y'(10) = 0.$$

Problem 8. Solve the following Sturm-Liouville problem:

$$y'' + 4y' + \lambda y = 0,$$

$$y(0) = 0, \qquad y(\pi) = 0.$$