MA 30300: Differential Equations And Partial Differential Equations For Engineering And The Sciences

Final Exam

December 18th, 2025

- The exam will be 120 minutes long.
- There are 20 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- For the Long Answer Questions, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. You may use any theorem proved in class provided you accurately state it before using it.
- For True or False (T/F) questions or Multiple Choice Questions (MCQs), you do not need to provide any justification.
- **Be sure to write your final answers in the designated spots.**
- Please use a writing instrument which is dark enough to be picked up by the scanner.
- When the time is over, all students must put down their writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ____________________

PUID: ____________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ____________________

QUESTION	VALUE	SCORE
1	15	
2	5	
3	5	
4	5	
5	5	
6	9	
7	6	
8	10	
TOTAL	60	

Table of Laplace Transforms and other formulae

Function $f(t)$	Laplace Transform $\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n for $n = 1, 2, 3 \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sinh(kt) = \frac{e^{kt}-e^{-kt}}{2}$	$\frac{k}{s^2-k^2}$
$\cosh(kt) = \frac{e^{kt}+e^{-kt}}{2}$	$\frac{s}{s^2-k^2}$
$e^{ct}f(t)$	$F(s-c)$
$f'(t)$	$sF(s) - f(0)$
$\int_0^t f(\tau)\, d\tau$	$\frac{F(s)}{s}$
$e^{at}f(t)$	$F(s-a)$
$-tf(t)$	$F'(s)$
$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma)d\sigma$
$(f * g)(t)$	$F(s)G(s)$
$u(t-a)f(t-a)$	$e^{-as}F(s)$

If $f(t)$ has Fourier series

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{L},$$

the Fourier coefficients are given by

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}dt, \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}dt.$$

The Dirac delta “function” $\delta(t)$ is defined by the property that for any $a \le b$ and any c,

$$\int_a^b f(t)\delta(t-c)\,dt = \begin{cases} f(c) & \text{if } a \le c \le b, \\ 0 & \text{otherwise.} \end{cases}$$

The equations

$$mx'' + kx = \cos\omega_0 t, \qquad my'' + ky = \sin\omega_0 t$$

with $\omega_0 = \sqrt{k/m}$ respectively have resonance solutions

$$x(t) = \frac{1}{2m\omega_0} t \sin\omega_0 t, \qquad y(t) = -\frac{1}{2m\omega_0} t\cos\omega_0 t.$$

The Euler update equations state $y_{j+1} = y_j + hk, \qquad x_{j+1} = x_j + h$, where k is an approximation to the average slope. In the case of the fourth-order Runge-Kutta method,

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

where

$$k_1 = f(x_j, y_j), k_2 = f(x_j + \tfrac{1}{2}h, y_j + \tfrac{1}{2}hk_1),$$

$$k_3 = f(x_j + \tfrac{1}{2}h, y_j + \tfrac{1}{2}hk_2), k_4 = f(x_{j+1}, y_j + hk_3).$$

The general solution to the wave equation

$$y_{tt} = a^2 y_{xx} \qquad (0 < x < L,\ t > 0),$$

under the constraints

$$y(0,t) = y(L,t) = 0,$$
$$y(x,0) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}, \qquad y_t(x,0) = 0,$$

is given by

$$y(x,t) = \sum_{n=1}^{\infty} b_n \cos\frac{n\pi a t}{L} \sin\frac{n\pi x}{L}.$$

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is given by

$$y(x,t) = \sum_{n=1}^{\infty} \frac{L B_n}{n\pi a} \sin\frac{n\pi a t}{L} \sin\frac{n\pi x}{L}.$$