MA 30300 (FALL 2025)

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SECTION WEB PAGE https://www.math.purdue.edu/~sahay5/fall2025/ma30300/index.html

NOTE : ALL IMAGES ARE FROM THE TEXTBOOK (EDWARDS, PENNEY, CALVIS 6th ed.)

ANNOUNCEMENTS / NOTES

1. CLASSES ARE BACK IN-PERSON NEXT WEEK

2. NO H.W. THE REST OF THIS WEEK

   ¡ PLEASE WORK ON H4 NOW ANYWAY ! (DUE : TUESDAY, SEPT 30th)

3. Θ 8 IS DUE MONDAY, SEPT 29th

4. RECORDING OF CLASS WILL BE UPLOADED TO KALTURA.

5. TRY TO KEEP VIDEOS ON.

REVIEW OF LAST CLASS

IVP

$$\boxed{\begin{aligned} x(0) &= 2 \\ x'(0) &= 1 \end{aligned}}$$

e.g. $x'' - x' - 6x = 0$

$\mathcal{L}\{x'\} = sX(s) - x(0) = sX(s) - 2$

$\mathcal{L}\{x''\} = s\mathcal{L}\{x'\} - x'(0) = s^2X(s) - sx(0) - x'(0)$

$= s^2X(s) - 2s - 1$

$\mathcal{L}$

$X(s) = \mathcal{L}\{x\}$

$(s^2 - s - 6)X(s) - 2s + 1 = 0$

$$\therefore X(s) = \frac{2s-1}{s^2-s-6} = \frac{2s-1}{(s-3)(s+2)}$$

$\Vert$ PARTIAL FRACTIONS

$$\frac{1}{s-3} + \frac{1}{s+2}$$

TABLE

$$\mathcal{L}\{e^{ct}\} = \frac{1}{s-c} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s-c}\right\} = e^{ct}$$

$$\therefore x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$= e^{3t} + e^{-2t}$$

e.g.

$$x'' + 9x = 5 \sin 2t \qquad x(0) = 0, \quad x'(0) = 0$$

$$\Downarrow \mathcal{L}$$

$$s^2 X(s) + 9X(s) = 5\left(\frac{2}{s^2+4}\right)$$

$$\Rightarrow X(s) = \frac{10}{(s^2+9)(s^2+4)} = \frac{2}{s^2+4} - \frac{2}{s^2+9}$$

(PARTIAL FRACTIONS)

TABLE: $\mathcal{L}\{\sin kt\} = \dfrac{k}{s^2+k^2}$

$$= \frac{2}{s^2+4} - \underbrace{\frac{2}{3}}_{=} \frac{3}{s^2+9}$$

$$\therefore\ x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s^2+2^2}\right\} + \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}$$

$$= \sin 2t + \frac{2}{3}\sin 3t$$

QUESTIONS?

Thm: (LAPLACE TRANSFORMS OF INTEGRALS)

$$\mathcal{L}\left\{ \underbrace{\int_0^t f(\tau)\,d\tau}_{\text{FUNCTION OF } t} \right\} = \frac{1}{s}\mathcal{L}\{f(t)\} = \frac{F(s)}{s}$$

PF IS VIA F.T. OF CALCULUS

USEFUL FOR THEORETICAL REASONS

ESSENTIALLY

$$\mathcal{L}\{f'\} \approx s\,\mathcal{L}\{f\}$$

$$\mathcal{L}\{\textstyle\int f\} \approx \frac{1}{s}\mathcal{L}\{f\}$$

$\boxed{x'' + 6x' + 34x = 0;}\quad x(0) = \underline{3}, \quad x'(0) = 1.$

$\mathcal{L}$

$\mathcal{L}\{x''\}$ & $\mathcal{L}\{x'\}$ IN TERMS OF $X(s) = \mathcal{L}\{x\}$

$\mathcal{L}\{x'\} = sX(s) - x(0) = sX(s) - 3$

$\mathcal{L}\{x''\} = s^2X(s) - sx(0) - x'(0) = s^2X(s) - 3s - 1$

$\mathcal{L}\{x''\} + 6\mathcal{L}\{x'\} + 34\mathcal{L}\{x\} = 0$

$s^2X(s) - 3s - 1 + 6\left(sX(s) - 3\right) + 34X(s) = 0$

$$-3s \qquad -1 \qquad\qquad -18$$

$$s^2 X(s) - 3s - 1 + 6\left(sX(s) - 3\right) + 34X(s) = 0$$

$$X(s)\left[s^2 + 6s + 34\right] - 3s - 19 = 0$$

$$\Rightarrow X(s) = \frac{3s + 19}{s^2 + 6s + 34} = \frac{3s + 19}{(s+3)^2 + 25}$$

$$s^2 + 6s + 34 = \left(s^2 + 6s + 9\right) + 25$$

$$(s+3)^2$$

$f(t)$	$F(s)$	
1	$\frac{1}{s}$	$(s > 0)$
t	$\frac{1}{s^2}$	$(s > 0)$
$t^n \quad (n \geqq 0)$	$\frac{n!}{s^{n+1}}$	$(s > 0)$
$t^a \quad (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}$	$(s > 0)$
e^{at}	$\frac{1}{s-a}$	$(s > a)$
$\cos kt$	$\frac{s}{s^2+k^2}$	$(s > 0)$
$\sin kt$	$\frac{k}{s^2+k^2}$	$(s > 0)$

$$3\left(\frac{(s+3)}{(s+3)^2+5^2}\right)$$

$$\frac{3s+19}{(s+3)^2+25} = \frac{3(s+3)}{(s+3)^2+25} + \frac{10}{(s+3)^2+25}$$

$$\frac{s}{s^2+k^2} \qquad \frac{k}{s^2+k^2} \qquad 2 \times \frac{5}{(s+3)^2+5^2}$$

TRANSLATED

TRANSLATED

$\underline{\text{Thm}}$:

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\Updownarrow$$

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at} f(t)$$

$(F(s) = \mathcal{L}\{f(t)\})$

$\rightarrow s > a + c$ WHERE $s > c$ IS THE DOMAIN OF F

$\underline{\text{Pf}}$: $\mathcal{L}\{e^{at} f(t)\} \underset{\text{DEFN.}}{=} \int_0^\infty \boxed{e^{-st} e^{at}} f(t)\, dt$

$\rightarrow e^{-(s-a)t}$

$$\int_0^\infty e^{-(s-a)t} f(t)\, dt = F(s-a)$$

$$X(s) = \frac{3(s+3)}{(s+3)^2+25} + \frac{10}{(s+3)^2+25}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = 3\,\mathcal{L}^{-1}\left\{\frac{(s+3)}{(s+3)^2+5^2}\right\} + 2\,\mathcal{L}^{-1}\left\{\frac{5}{(s+3)^2+5^2}\right\}$$

$$3\,\underbrace{\mathcal{L}^{-1}\{F(s+3)\}}_{\text{THM} \downarrow\ e^{-3t}\cos 5t} + 2\,\underbrace{\mathcal{L}^{-1}\{G(s+3)\}}_{e^{-3t}\sin 5t}$$

$f(t)$	$F(s)$	
1	$\frac{1}{s}$	$(s>0)$
t	$\frac{1}{s^2}$	$(s>0)$
$t^n \quad (n \geqq 0)$	$\frac{n!}{s^{n+1}}$	$(s>0)$
$t^a \quad (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}$	$(s>0)$
e^{at}	$\frac{1}{s-a}$	$(s>a)$
$\cos kt$	$\frac{s}{s^2+k^2}$	$(s>0)$
$\sin kt$	$\frac{k}{s^2+k^2}$	$(s>0)$

$$\mathcal{L}\{\cos 5t\} = \frac{s}{s^2+5^2} = F(s)$$

$$\mathcal{L}\{\sin 5t\} = \frac{5}{s^2+5^2} = G(s)$$

$$\Rightarrow \quad x(t) = 3e^{-3t}\cos 5t + 2e^{-3t}\sin 5t$$

UPSHOT: KEEP TRANSLATIONS OF THE LHS OF LAPLACE TRANSFORM TABLES IN YOUR MIND.

$f(t)$	$F(s)$	
$\boxed{1}$	$\boxed{\frac{1}{s}}$	$(s>0)$
t	$\frac{1}{s^2}$	$(s>0)$
$t^n \quad (n \geqq 0)$	$\frac{n!}{s^{n+1}}$	$(s>0)$
$t^a \quad (a>-1)$	$\frac{\Gamma(a+1)}{s^{a+1}}$	$(s>0)$
$\boxed{e^{at}}$	$\boxed{\frac{1}{s-a}}$	$(s>a)$
$\cos kt$	$\frac{s}{s^2+k^2}$	$(s>0)$
$\sin kt$	$\frac{k}{s^2+k^2}$	$(s>0)$

$n=2.$

n, a, k

$$\frac{1}{s-a} \xleftarrow{\text{TRANSLATE BY } a} \frac{1}{s}$$

$$e^{at} \xleftarrow{\text{MULTIPLY BY } a} 1$$

$f(t)$	$F(s)$	
$e^{at}t^n$	$\dfrac{n!}{(s-a)^{n+1}}$	$(s>a)$
$e^{at}\cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$	$(s>a)$
$e^{at}\sin kt$	$\dfrac{k}{(s-a)^2+k^2}$	$(s>a)$

$=$

$\dfrac{n!}{s^{n+1}}$

$\dfrac{s}{s^2+k^2}$

$\dfrac{k}{s^2+k^2}$

$$y'' + 4y' + 4y = t^2; \quad y(0) = y'(0) = 0.$$

$\mathcal{L}\{y\} = Y(s)$

$\mathcal{L}\{y'\} = sY(s)$

$\mathcal{L}\{y''\} = s^2 Y(s)$

$\mathcal{L}$

$$\underbrace{s^2 Y(s) + 4sY(s) + 4Y(s)}_{(s^2+4s+4)Y(s)} = \mathcal{L}\{t^2\} = \frac{2}{s^3}$$

$$Y(s) = \frac{2}{s^3(s^2+4s+4)} = \frac{2}{s^3(s+2)^2}$$

PARTIAL FRACTIONS $\dfrac{2}{s^3(s+2)^2}$

$f(t)$	$F(s)$	
1	$\boxed{\dfrac{1}{s}}$	$(s > 0)$
$\frac{1}{2}\times($ t	$\dfrac{1}{s^2}$ $)$	$(s > 0)$
t^n $(n \geqq 0)$ t^2	$\dfrac{n!}{s^{n+1}}$ $\dfrac{2}{s^3}$	$(s > 0)$
t^a $(a > -1)$	$\dfrac{\Gamma(a+1)}{s^{a+1}}$	$(s > 0)$
e^{at}	$\dfrac{1}{s-a}$	$(s > a)$
$\cos kt$	$\dfrac{s}{s^2+k^2}$	$(s > 0)$
$\sin kt$	$\dfrac{k}{s^2+k^2}$	$(s > 0)$

$n=2$

$a=-2$

① ② ③ ④ ⑤

$$Y(s) = \frac{\frac{1}{2}}{s^3} - \frac{\frac{1}{2}}{s^2} + \frac{\frac{3}{8}}{s} - \boxed{\frac{\frac{1}{4}}{(s+2)^2}} - \frac{\frac{3}{8}}{s+2},$$

$\mathcal{L}^{-1} = \frac{1}{4}e^{-2t}t$

$\underbrace{\frac{1}{4}\cdot\frac{2}{s^3}}$ $\xrightarrow{\mathcal{L}^{-1}}$ $\frac{t^2}{4}$

$\xrightarrow{\mathcal{L}^{-1}}$ $\frac{1}{2}t$

$\xrightarrow{\mathcal{L}^{-1}}$ $3/8$

$\xrightarrow{\mathcal{L}^{-1}}$ $\frac{3}{8}e^{-2t}$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\} = e^{-2t}t$

$$x(t) = \frac{t^2}{4} + \frac{t}{2} + \frac{3}{8} - \frac{t e^{-2t}}{4} - \frac{3}{8} e^{-2t}$$