MA 30300: Differential Equations And Partial Differential Equations For Engineering And The Sciences

Midterm 1

October 6th, 2025

- This is the exam for §764 of MA 303.
- The exam will be 60 minutes long.
- There are 16 pages.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use theorems proved in class provided you accurately state them before using them.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- When the time is over, put down your writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ____________________________________

PUID: ____________________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ____________________________________

QUESTION	VALUE	SCORE
1	15	
2	5	
3	5	
4	5	
5	3	
6	7	
TOTAL	40	

Table of Laplace Transforms

Function $f(t)$	Laplace Transform $\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n for $n = 1, 2, 3\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sinh(kt) = \frac{e^{kt}-e^{-kt}}{2}$	$\frac{k}{s^2-k^2}$
$\cosh(kt) = \frac{e^{kt}+e^{-kt}}{2}$	$\frac{s}{s^2-k^2}$
$e^{ct}f(t)$	$F(s-c)$
$f'(t)$	$sF(s) - f(0)$
$\int_0^t f(\tau)\, d\tau$	$\frac{F(s)}{s}$
$e^{at}f(t)$	$F(s-a)$
$-tf(t)$	$F'(s)$
$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma) d\sigma$
$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$

1. (15 points)

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) If the system $d\vec{x}/dt = A\vec{x}$ has an isolated critical point at $\vec{x} = (0, 0)$, then the eigenvalues of the matrix A must be nonzero.

(b) The Jacobian of the system $x' = x^2 - xy$ and $y' = y^2 - xy$ at the point $(-1, -1)$ is $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

(c) If the eigenvalues of a 2×2 matrix A are purely imaginary, then the critical point of $x' = Ax$ is a stable spiral point.

(d) If a linear system $x' = ax + by$ and $y' = cx + dy$, $a, b, c, d \in \mathbb{R}$ has purely imaginary eigenvalues, then the critical points of a linear perturbation of this system will be stable but not asymptotically so.

(e) For the competition system $x' = x - 2x^2 - 2xy, y' = 3y - 3y^2 - 2xy$, one of the two species will die out except at the co-existence point.

(f) If a 4×4 matrix A has a generalized eigenvector of rank 4, then A must have an eigenvalue of algebraic multiplicity 4 and geometric multiplicity 1.

(g) If $f(t)$ and $g(t)$ are continuous functions such that for some c, $F(s) = G(s)$ for all $s > c$, then $f(t) = g(t)$ for every value of t.

(h) For any function $f(t)$ with $f(0) = -1$, $\mathcal{L}\{f'(t)\} = sF(s) - 1$.

(i) If λ_1 and λ_2 are the eigenvalues of a linear system with two dependent variables and $\mathrm{Re}(\lambda_1) < 0$ and $\mathrm{Re}(\lambda_2) < 0$, then the critical point is a sink, and hence is asymptotically stable.

(j) The critical point of a linear system can be unstable only if it is a nodal source.

2. (5 points)

For the following two statements determine whether they are true or false. In either case, provide appropriate justification for your answer (e.g., a counterexample or a brief proof).

(a) If $F(s) = \mathcal{L}\{f(t)\}$, then
$$\mathcal{L}\{-tf'(t)\} = sF'(s) + F(s).$$

(b) If A is a 2×2 matrix with complex entries, with complex eigenvalues λ_1 and λ_2, then λ_1 is the complex conjugate of λ_2.

(Continued)

3. (5 points) Find the Laplace transform of the function

$$f(t) = te^{-3t} \sin 2t$$

Final Answer:

(Continued)

4. (5 points) Find the general solution $\vec{x}(t) = (x_1, x_2)$ to the system of equations

$$x_1' = 3x_1 + x_2,$$
$$x_2' = -x_1 + x_2.$$

Final Answer:

(Continued)

5. (3 points) The following autonomous system has only one critical point. Find it and determine the type and stability of the point.

$$\frac{dx}{dt} = x + y^2,$$
$$\frac{dy}{dt} = 2 - y.$$

Final Answer:

(Continued)

6. (7 points) Solve the following initial value problem using Laplace transforms:

$$x'' + 4x = (16t + 8)e^{2t},$$
$$x(0) = 0, \qquad x'(0) = 0.$$

Final Answer:

(Continued)

Scratch Work

Scratch Work