Problem Solving Seminar

AMS Grad Student Chapter, University of Rochester

6th March, 2020

Problem 1 (+++*, MOP Test 2008/7/2)**.** Suppose that a, b, c are positive real numbers such that for every integer n,

$$\lfloor an \rfloor + \lfloor bn \rfloor = \lfloor cn \rfloor$$

Prove that at least one of a, b, c is an integer. [1]

Problem 2 (??, Problem 1.55 from [GP14])**.** Suppose that X is a metric space, Y is a complete metric space, $D \subseteq X$ is a nonempty set and $f : D \to Y$ is a continuous function. Show that f can be extended continuously to a G_δ-subset of X containing D.

Problem 3 (+, Chapter 2, Problem 39 from [KT06])**.** Two players alternately choose uncountable subsets $K_0 \supset K_1 \supset \cdots$ of the real line. Show that no matter how the first player plays, the second one can always achieve

$$\bigcap_{n=0}^{\infty} K_n = \emptyset$$

.

*+ indicates hardness; the more plusses there are, the harder I think the problem is. Conversely, − indicates easiness. ? indicates I don't know the solution, and the number of ?s indicates how hard I think the solution probably is.

References

[GP14] L. Gasińksi and N.S. Papageorgiou. *Exercises in Analysis*. Number pt. 1 in Problem Books in Mathematics. Springer International Publishing, 2014.

[KT06] Péter Komjáth and Vilmos Totik. *Problems and theorems in classical set theory*. Springer Science & Business Media, 2006.

Hints

1. If x is irrational, then $\{xn\} = xn - \lfloor xn \rfloor$ is equidistributed in $[0, 1)$. Sample n uniformly at random from $\{1, \cdots, N\}$ and take $N \to \infty$.