Homework 3

MA 35100 (Spring 2024, Section 130)

January 26th, 2024 (updated: Feb 1st)

Instructions

- Due: (~~Friday, February 2nd~~) Saturday, February 3rd at 11 PM Eastern Time.
- Total Score: 25 points.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [9 points] For the following sets, determine if they are are linearly dependent.

1. $V_1 = P_2(\mathbb{R})$, $S_1 = \{t, t^2\}$.
2. $V_2 = \mathbb{R}^3$, $S_2 = \{(1,0,0), (2,3,1), (0,0,1), (7,8,100)\}$.
3. $V_3 = M_{2\times 2}(\mathbb{R})$, $S_3 = \{A_1, A_2, A_3\}$, with

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Problem 2. [10 points] Given a vector space V, a set S of elements in V is called *spanning* if $V = \operatorname{Span} S$. For the V_j and S_j in Problem 1, determined if S_j is spanning. Further, if S_j is not spanning, then find an alternative description for $\operatorname{Span} S_j$.

Problem 3. [4 points] For the S_j in Problem 1 which are not spanning, find a set S'_j (as small as possible) such that $S_j \subset S'_j$ (i.e., S'_j contains all elements in S_j plus a few more) such that S'_j is spanning.

Problem 4. [2 points] For the S_j in Problem 1 which are not linearly independent, find a set S''_j (as large as possible) such that $S''_j \subset S_j$ (i.e., S''_j is obtained by deleting elements from S_j) such that S''_j is linearly independent.