Homework 5

MA 35100 (Spring 2024, Section 130)

February 11th, 2024

Instructions

- Due: Saturday, February 17th at 11 PM Eastern Time.
- Total Score: 25 points.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

The following is relevant for Problems 1, 2, 3, and 4:

$$A_1 = \begin{bmatrix} 1 & 2 & 0 & -2 \\ 2 & 3 & 2 & 3 \\ 2 & 10 & 4 & 10 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 5 & 0 & 1 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 3 & 6 & 6 \\ -2 & -4 & -4 \end{bmatrix}.$$

Problem 1. [3 points] For each $A \in \{A_1, A_2, A_3\}$, what is the dimension of the ambient Euclidean space in which $\operatorname{Col} A$, $\operatorname{Row} A$, and $\operatorname{Null} A$ live?

Problem 2. [6 points] For each $A \in \{A_1, A_2, A_3\}$, find a basis for $\operatorname{Col} A$.

Problem 3. [6 points] For each $A \in \{A_1, A_2, A_3\}$, find a basis for $\operatorname{Row} A$. Verify that $\dim(\operatorname{Col} A) = \dim(\operatorname{Row} A) = \operatorname{Rank}(A)$.

Problem 4. [6 points] or each $A \in \{A_1, A_2, A_3\}$, find a basis for $\operatorname{Null} A$. Verify that the rank-nullity theorem holds for A.

Problem 4. [4 points] For the following spaces V, determine $\dim V$.

1. $V = \{A \in M_{3\times 3}(R) : A^T = -A\}$.
2. $V = \{p(t) \in P_3(\mathbb{R}) : p'(0) = 0 \text{ and } p(1) = 0\}$.