Homework 8

MA 35100 (Spring 2024, Section 130)

March 6th, 2024 (updated: March 20th)

Instructions

- Due: Saturday, March 23rd at 11 PM Eastern Time.
- Total Score: 25 points.
- The three lowest homework scores will be dropped from the final grade.
- One late submission is permitted (over the course of the semester) with no questions asked.
- Submissions can be hand-written or typed in LaTeX and must be submitted on Gradescope.
- You are allowed to discuss and collaborate on problems. However, each student must work on the final submission on their own. **In particular, copying someone else's final submission will be considered cheating and will be reported to the Office of the Dean of Students.**

Problem 0. [0 points] Copy paste the following text in the beginning of your submission:

> I have not made use of any unauthorized resources (including online resources) while working on this submission. Any collaboration with other students conforms with the policies of this course.

After that, list all students you collaborated with, clearly indicating which problems you worked with them on. If you did not collaborate with anyone, clearly state this instead.

Problem 1. [5 points] Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear map satisfying

$$T\left(\begin{bmatrix}1\\2\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix} \qquad \text{and} \qquad T\left(\begin{bmatrix}2\\1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\2\end{bmatrix}.$$

Is this enough information to determine $T(5, -2, 1)$? What about $T(5, 0, 1)$? Explain this phenomenon.

Problem 2. [5 points] Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the map $T(x, y, z) = (x + y, x + z, y + z)$. Show that T is an invertible linear transformation, and compute its inverse. Further, find the matrix of T and the matrix of T^{-1} and check that they are inverses of each other (as matrices).

Problem 3. [5 points] Let $S : \mathbb{R}^3 \to \mathbb{R}^2$ be the map $S(x, y, z) = (x + y + z, x - y)$. Show that S is a linear transformation. Compute the composition $S \circ T$ – what is its domain and co-domain? Further, compute the matrices of S, T, and $S \circ T$ and verify that they respect the connection between compositions of transformations and matrix multiplication.

Problem 4. [10 points] For each of the linear transformations $T_j : \mathbb{R}^2 \to \mathbb{R}^2$, where T_j, $j = 1, 2, 3, 4$ are given below, compute the standard matrix of T_j.

- $T_1(x, y) = (x, 0)$.
- T_2 is an anticlockwise rotation around the origin by an angle of $\pi/4$ radians.
- $T_3(x, y) = (x, 2y)$.
- T_4 is a reflection along the line $x + y\sqrt{2} = 0$.