Question Bank for Final Exam

MA 35100 (Spring 2024, Section 130)

April 28th, 2024

Notes

- There will be three types of questions on the final exam: T/F (True or False), MCQs (Multiple Choice Questions), and Long Answer Questions (LAQs).
- This is a sample question bank for LAQs.
- For samples of T/F and MCQs, please see the exam archives.
- The structure of the final exam will be as follows:
  - 10 T/Fs (2 points each = 20 points total).
  - 4 MCQs (5 points each = 20 points total). Each MCQ will have 5 options, of which only one is correct.
  - 4 LAQs (6-10 points each depending on complexity = 30 points total)
- The final exam will be worth 70 points total.

Long Answer

Problem 1. Compute the LU decomposition of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 7 & 11 \\ -1 & -4 & -9 & -16 & -25 \\ 0 & 0 & 4 & 8 & 12 \end{bmatrix}$$

Problem 2. Find all (complex) solutions of the following system of equations:

$$\begin{array}{cccccc} iz_1 & +z_2 & -(1+i)z_3 & = & 1+4i \\ 3z_1 & +iz_2 & & = & 3 \\ -z_1 & -z_2 & +(2+i)z_3 & = & 2+4i \end{array}$$

Problem 3. Diagonalize the following matrix:

$$\begin{bmatrix} 0 & 5 & 3 \\ -3 & 8 & 3 \\ 7 & -13 & -4 \end{bmatrix}.$$

Problem 4. Consider the sequence given by $a_0 = 0$, $a_1 = 1$, and

$$a_n = 2a_{n-1} + a_{n-2} \qquad \text{for } n \geq 2.$$

Find a formula for a_n.

Problem 5. Solve the following initial value problem:

$$\begin{aligned} x_1' &= 2x_1 + x_2, \\ x_2' &= -x_1 + 2x_2, \end{aligned}$$

where

$$\begin{aligned} x_1(0) &= 0, \\ x_2(0) &= 1. \end{aligned}$$

Problem 6. Solve the following initial value problem:

$$y'' = 5y' - 6y,$$

$$y(0) = 0, \qquad y'(0) = -1.$$

Problem 7. Consider the sequence given by $a_0 = 1$, $a_1 = 3$, and

$$a_n = 5a_{n-1} - a_{n-2} \qquad \text{for } n \geq 2.$$

Find a formula for a_n.

Problem 8. Consider the set $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4\}$ given by

$$\vec{b}_1 = (1,1,1,1), \qquad \vec{b}_2 = (1,-1,1,-1), \qquad \vec{b}_3 = (1,1,-1,-1), \qquad \vec{b}_4 = (1,-1,-1,1).$$

(a) Find $b_i \cdot b_j$ for every i, j. (Don't forget the case $i = j$).

(b) Why can you conclude that $\mathcal{B}$ is a basis of $\mathbb{R}^4$?

(c) Find $[\vec{x}]_{\mathcal{B}}$, where $\vec{x} = (4, 8, 0, -12)$.

Problem 9. Find the general solution $y(t)$ to the ODE

$$y^{(3)} = 2y^{(2)} + 5y' - 6y.$$

Problem 10. Consider the sequence given by $a_0 = 1$, $a_1 = 3$, $a_2 = 9$ and

$$a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3} \qquad \text{for } n \geq 3.$$

Find a formula for a_n.