MA 35100 (Sec: 130): Elementary Linear Algebra

Midterm 1

February 20th, 2024

- The exam will be 60 minutes long.
- There are 15 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use any theorem proved in class provided you accurately state it before using it.
- When the time is over, all students must put down their writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ______________________________

PUID: ______________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	15	
2	5	
3	5	
4	5	
5	5	
6	5	
TOTAL	40	

1. (15 points)

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers (however, please see the next question).

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) The dimension of the space $P_3(\mathbb{R})$ is 3.

(b) If A is an $m \times n$ matrix with $m < n$, then the system $A\vec{x} = \vec{b}$ has either no solutions or infinitely many solutions.

(c) If the set $\{\vec{x}, \vec{y}, \vec{z}\}$ of vectors in $\mathbb{R}^3$ is linearly independent, then it must be a basis.

(d) The pivot columns of a matrix A are the columns in the row-reduced echelon form of A which have the pivots in them.

(e) The row-space and column-space of a symmetric matrix are equal.

(f) If A and B are row-equivalent, then they have the same column-space.

(g) If A and B are row-equivalent, then they have the same nullspace.

(h) The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in row-reduced echelon form.

(i) The vector $(2, 0, 4, 10)$ is a linear combination of the vectors $(1, 2, 0, 3)$ and $(0, -1, 1, 1)$.

(j) For a fixed vector $\vec{x} \in \mathbb{R}^n$, the set of all $m \times n$ matrices, B such that $B\vec{x} = \vec{0}$ is a subspace of $M_{m\times n}(\mathbb{R})$.

2. (5 points)

Pick one true statement and one false statement in Q1 and provide an explanation for why these are true or false. Your explanation should either be a counterexample or a brief justification using theorems proved in class. Please make sure it is clear which parts you have chosen to discuss.

(Continued)

3. (5 points) For what values of k does the following system have no solution, a unique solution, and infinitely many solutions respectively?

$$
\begin{aligned}
x + y - z &= 3 \\
2x + y + z &= 0 \\
x + (k^2 - 7)z &= k
\end{aligned}
$$

Final Answer:

(Continued)

4. (5 points) Find all solutions to the following system of equations.

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 - x_2 - x_3 - x_4 &= -1 \\ 3x_1 + 7x_2 - x_3 - 9x_4 &= 0 \\ x_2 + x_3 + x_4 &= 3 \end{aligned}$$

Final Answer:

(Continued)

5. (5 points)

Find a basis for Span S, where

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 7 \\ 11 \\ 13 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix} \right\}.$$

Final Answer:

(Continued)

6. (5 points) Compute the rank and nullity of the following matrix:

$$\begin{bmatrix} -3 & -1 & 3 & -3 & 5 \\ 0 & 1 & 0 & 2 & 10 \\ 3 & 2 & -3 & 5 & 5 \\ -4 & 0 & 4 & -8 & 0 \end{bmatrix}$$

Final Answer:

(Continued)

Scratch Work

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