MA 35100 (Sec: 130): Elementary Linear Algebra

Midterm 2

April 2nd, 2024

- The exam will be 60 minutes long.
- There are 14 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use any theorem proved in class provided you accurately state it before using it.
- When the time is over, all students must put down their writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ______________________________

PUID: ______________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	15	
2	5	
3	5	
4	5	
5	5	
6	5	
TOTAL	40	

1. (15 points)

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers (however, please see the next question).

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) If A is a 2×3 matrix and B is a 3×2 matrix, then $\det AB = 0$.

(b) If A and B are both $n \times n$ matrices, then $(AB)^T = A^T B^T$.

(c) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation. Then T is not injective.

(d) The linear map $T : \mathbb{R}^n \to \mathbb{R}^n$ is an isomorphism if and only if the matrix of T is invertible.

(e) If $\mathcal{B} = \{\vec{b}_1, \cdots, \vec{b}_n\}$ is an ordered basis for $\mathbb{R}^n$, then the coordinate matrix $C_{\mathcal{B}}$ has the basis element $\vec{b}_j$ as its jth column.

(f) A square matrix with two equal rows has determinant 1.

(g) If $A^{-1} = A^T$ for a invertible matrix A, then $\det A = 1$.

(h) If P is a parallelogram in $\mathbb{R}^2$ and $A(P)$ is the parallelogram resulting from an application of the matrix transformation A on the parallelogram P, then the area of $A(P)$ is $|\det A|$ times the area of P.

(i) If A is an $n \times n$ matrix such that the system of equations $A\vec{x} = \vec{b}$ always has a solution for every $\vec{b} \in \mathbb{R}^n$, then $\det A \neq 0$.

(j) The image of a matrix transformation is the same as its column space.

2. (5 points)

Pick one true statement and one false statement in Q1 and provide an explanation for why these are true or false. Your explanation should either be a counterexample or a brief justification using theorems proved in class. Please make sure it is clear which parts you have chosen to discuss.

(Continued)

3. (5 points) Suppose that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -10.$$

Compute the following determinant.

$$\begin{vmatrix} a-c-d+f & d-f & g-i \\ 3c-3f & 3f & 3i \\ b/5-e/5 & e/5 & h/5 \end{vmatrix}$$

Final Answer:

(Continued)

4. (5 points) Compute the inverse of the following matrix.

$$A = \begin{bmatrix} -2 & -2 & -3 & 4 \\ 0 & 1 & 1 & -1 \\ 5 & 6 & 8 & -10 \\ 3 & 1 & 3 & -4 \end{bmatrix}$$

Final Answer:

(Continued)

5. (5 points)

Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ be an ordered basis of $\mathbb{R}^2$ with

$$\vec{b}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \qquad \vec{b}_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}.$$

Compute the point matrix $P_{\mathcal{B}}$ and the coordinate matrix $C_{\mathcal{B}}$. Further, for

$$\vec{x} = \begin{bmatrix} 11 \\ 0 \end{bmatrix}, \qquad [\vec{y}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -2 \end{bmatrix},$$

compute $[\vec{x}]_{\mathcal{B}}$ and $\vec{y}$.

Final Answer:

(Continued)

6. (5 points) Consider the linear map $T : P_3(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ given by

$$T(p(t)) = \begin{bmatrix} p(0) & p'(1) \\ p(-1) & p''(0) \end{bmatrix}.$$

Given the ordered bases $\mathcal{B}_1 = \{1, t, t^2, t^3\}$ of $P_3(\mathbb{R})$ and $\mathcal{B}_2 = \{E_{11}, E_{12}, E_{21}, E_{22}\}$ of $M_{2\times 2}(\mathbb{R})$, find the matrix of the transformation T in these bases. Is this map an isomorphism?

Final Answer:

(Continued)

Scratch Work