Question Bank for Final Exam

MA 35100 (Spring 2025, §§130–131)

April 28th, 2025

Notes

- There will be three types of questions on the final exam: T/F (True or False), MCQs (Multiple Choice Questions), and Long Answer Questions (LAQs).
- This is a sample question bank for LAQs.
- For samples of T/F and MCQs, please see the exam archives.
- This bank focuses on material after Midterm 2. LAQs similar to the ones that showed up in Midterm 1 and Midterm 2 may also show up in the final; for these also, please see the archives.
- The structure of the final exam will be as follows:
  - 10 T/Fs (2 points each = 20 points total).
  - 4 MCQs (5 points each = 20 points total). Each MCQ will have 5 options, of which only one is correct.
  - 4 LAQs (6-10 points each depending on complexity = 30 points total)
- The final exam will be worth 70 points total.

Long Answer

Problem 1. Compute the LU decomposition of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 7 & 11 \\ -1 & -4 & -9 & -16 & -25 \\ 0 & 0 & 4 & 8 & 12 \end{bmatrix}$$

Problem 2. Find all (complex) solutions of the following system of equations:

$$\begin{array}{rrrcr} iz_1 & +z_2 & -(1+i)z_3 & = & 1+4i \\ 3z_1 & +iz_2 & & = & 3 \\ -z_1 & -z_2 & +(2+i)z_3 & = & 2+4i \end{array}$$

Problem 3. Diagonalize the following matrix:

$$\begin{bmatrix} 0 & 5 & 3 \\ -3 & 8 & 3 \\ 7 & -13 & -4 \end{bmatrix}.$$

Problem 4. Find the following determinant:

$$\begin{vmatrix} i & 1+i & 1 & 1-i \\ 1 & 1 & 1 & 1 \\ -1 & 2i & 1 & -2i \\ -i & -2+2i & 1 & -2-2i \end{vmatrix}$$

Problem 5. Solve the following initial value problem:

$$\begin{aligned} x_1' &= 2x_1 + x_2, \\ x_2' &= -x_1 + 2x_2, \end{aligned}$$

where

$$\begin{aligned} x_1(0) &= 0, \\ x_2(0) &= 1. \end{aligned}$$

Problem 6. Solve the following initial value problem:

$$y'' = 5y' - 6y,$$

$$y(0) = 0, \qquad y'(0) = -1.$$

Problem 7. Find the general solution $y(t)$ to the ODE

$$y^{(3)} = 2y^{(2)} + 5y' - 6y.$$

Problem 8. Find the general solution to the following system of ODEs:

$$\begin{aligned} x_1' &= x_1 + 2x_2, \\ x_2' &= -2x_1 + x_2. \end{aligned}$$

Your answer should be purely real (i.e., no complex-valued functions).

Problem 9. Suppose a graph has the following adjacency matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Draw a diagram of the graph. Compute its walk matrix M.

(b) Find the eigenvalues and eigenvectors of M, and hence diagonalize it.

(c) Using your answer, derive a formula for the state vector at time $t \in \mathbb{N}$ for a walk that starts at vertex 1 at time $t = 0$.

(d) Is this is a mixing random walk? Explain your answer.

Problem 10. Suppose a graph has the following adjacency matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

(a) Draw a diagram of the graph. Compute its walk matrix M.

(b) Find the eigenvalues and eigenvectors of M, and hence diagonalize it.

(c) Using your answer, derive a formula for the state vector at time $t \in \mathbb{N}$ for a walk that starts at vertex 2 at time $t = 0$.

(d) Is this is a mixing random walk? Explain your answer.

Problem 11. Suppose a graph has the following adjacency matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

(a) Draw a diagram of the graph. Compute its walk matrix M.

(b) Find the eigenvalues and eigenvectors of M, and hence diagonalize it.

(c) Using your answer, derive a formula for the state vector at time $t \in \mathbb{N}$ for a walk that starts with a randomized starting position with probability distribution $(\frac{1}{2}, \frac{1}{2}, 0)$.

(d) Is this is a mixing random walk? Explain your answer.