MA 35100: Elementary Linear Algebra

Final Exam

May 5th, 2025

- The exam will be 120 minutes long.
- There are 19 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- For the Long Answer Questions, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. You may use any theorem proved in class provided you accurately state it before using it.
- For True or False (T/F) questions or Multiple Choice Questions (MCQs), you do not need to provide any justification.
- **Be sure to write your final answers in the designated spots.**
- Please use a writing instrument which is dark enough to be picked up by the scanner.
- When the time is over, all students must put down their writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ______________________________

PUID: ______________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	20	
2	5	
3	5	
4	5	
5	5	
6	6	
7	9	
8	6	
9	9	
TOTAL	70	

True or False

1. (20 points)

Determine if the following statements are true or false. Fill in your answers in the associated box. You do not need to justify your answers.

(a) If A is diagonalizable over $\mathbb{C}$, then $\det A \neq 0$.

(a):

(b) If $A \in M_n(\mathbb{C})$ has eigenvalue -1, then it cannot satisfy $A^3 = I_n$.

(b):

(c) A linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^3$ cannot be injective.

(c):

(d) A matrix $A \in M_n(\mathbb{R})$ is nonsingular if and only if $\operatorname{Rank} A = n$ which happens if and only if $\det A \neq 0$.

(d):

(e) If $A^T = A$, then A is invertible.

(e):

(f) Let $C = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ be the unit circle. For any linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$, the image $T(C)$ must also be a circle.

(f):

(g) If A and B are row-equivalent, then $\operatorname{Row}(A) = \operatorname{Row}(B)$ and $\operatorname{Col}(A) = \operatorname{Col}(B)$.

(g):

(h) If a matrix in $M_n(\mathbb{C})$ is deficient, then it must have a repeated eigenvalue in the sense of algebraic multiplicity.

(h):

(i) The matrix $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ could be the adjacency matrix of a simple graph.

(i):

(j) If n, r, and $r^{\#}$ have their usual meanings for a linear system, then $r, r^{\#} \leqslant n$.

(j):

Multiple Choice Questions

2. (5 points)

Which of the following statements is true?

(a) $\mathbb{R}^2$ is a subspace of $\mathbb{C}^2$ when considered as a vector space over $\mathbb{C}$.

(b) The dimension of $\mathbb{C}^2$ as a vector space over $\mathbb{R}$ is 2.

(c) The set $\{(z_1, z_2, z_3) \in \mathbb{C}^3 : z_1 - z_2 + iz_3 = 1\}$ is a subspace of $\mathbb{C}^3$.

(d) The set $\{(z_1, z_2, z_3) \in \mathbb{C}^3 : z_1 - z_2 + iz_3 = 0\}$ is a subspace of $\mathbb{C}^3$.

(e) The dimension of $P_2(\mathbb{C})$ as a vector space over $\mathbb{C}$ is 2.

Answer:

3. (5 points)

Which of the following matrices is *not* diagonalizable over $\mathbb{C}$?

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$.

(e) $\begin{bmatrix} 2026 & 1 & 0 \\ 0 & 2025 & 0 \\ 0 & 1 & 2025 \end{bmatrix}$

Answer:

4. (5 points)

Let M_{ij} be the (i, j)th minor and C_{ij} be the (i, j)th cofactor of the following matrix:

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

Which of the following is correct?

(a) $M_{23} = 4$ and $C_{14} = 4$.

(b) $M_{23} = -4$ and $C_{14} = 4$.

(c) $M_{23} = 4$ and $C_{14} = -4$.

(d) $M_{23} = -4$ and $C_{14} = -4$.

(e) None of the above.

Answer:

5. (5 points)

Let $V = \{p \in P_3(\mathbb{R}) : p(0) = p'(0) = 0\}$ be the subspace of P_3 consisting of polynomials with $p(0) = p'(0) = 0$. Which of the following sets is a basis for V?

(a) $\{t^3, t^4\}$.

(b) $\{t^2, t^3\}$.

(c) $\{1, t, t^2, t^3\}$.

(d) $\{1 + t, t^2 - t^3\}$.

(e) $\{t + t^2, t^3\}$.

Answer:

Long Answer Questions

6. (6 points)

Compute the LU decomposition of the following matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 2 & 1 \\ -2 & -1 & 5 & 0 & 2 \\ 1 & 1 & 0 & 2 & 3 \\ 0 & 0 & 4 & 5 & 4 \end{bmatrix}$$

$L =$	$U =$

(Continued)

7. (9 points)

Compute the following determinant.

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1+i & 2i & -4 & 1+i & 0 \\ i & -i & i & -i & i \\ 1 & 1+i & -2+2i & 1 & 0 \end{vmatrix}$$

Final Answer:

(Continued)

8. (6 points)

Solve the following initial value problem:

$$\begin{aligned} x_1' &= 5x_1 + 2x_2, \\ x_2' &= 2x_1 + 5x_2 \end{aligned}$$

$$x_1(0) = 4, \quad x_2(0) = 2.$$

Final Answer:

(Continued)

9. (9 points)

Suppose a graph has the following adjacency matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

(a) Draw a diagram of the graph with the vertices clearly labeled:

(b) Compute its walk matrix M.

$M =$

(c) Find the eigenvalues and eigenvectors of M, and hence write $M = P\Delta P^{-1}$.

$P =$

$\Delta =$

(d) Derive a formula for the state vector at time $t \in \mathbb{N}$ for a walk that starts at vertex 3.

$\vec{p}(t) =$

(e) True or false: this random walk is mixing. Explain your answer.

(Continued)

Scratch Work

Scratch Work

Scratch Work