MA 35100 (SPRING 2025, SEC: 130)

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SECTION WEBPAGE https://www.math.purdue.edu/~sahay5/spring2025/ma35100/index.html

NOTE: ALL IMAGES ARE FROM THE TEXTBOOK (R.C. PENNEY, 4th ed.)

ANNOUNCEMENTS / NOTES

1. CLASSES ARE BACK IN-PERSON FROM TUESDAY.

2. NO OFFICE HOURS OR H.W. THIS WEEK

   ¡ PLEASE WORK ON H.W. 2 NOW ANYWAY ! (DUE: WED, FEB 5th)

3. RECORDING OF CLASS WILL BE UPLOADED TO KALTURA.

4. TRY TO KEEP VIDEOS ON.

DEFN: A SET S IS LINEARLY INDEPENDENT (L.I.) IF NONE OF THE ELEMENTS IS A LINEAR COMBINATION OF THE OTHERS.

CONVERSELY, A SET IS LINEARLY DEPENDENT (L.D.) IF ONE OF ITS ELEMENTS IS A LINEAR COMBINATION OF THE OTHERS.

e.g. 1 ~~L.D.~~

$$\left\{ \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

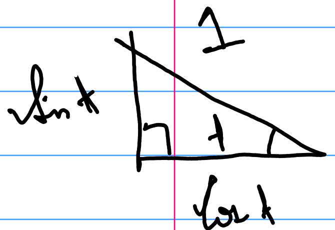
e.g. 2 L.I.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \boxed{0} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \boxed{0} \\ 0 \\ 1 \end{bmatrix} \right\}$$

e.g 3

$\{ 1, \sin^2 t, \cos^2 t \}$ $\quad V = \mathcal{F}(\mathbb{R})$

$\sin^2 t + \cos^2 t = 1$ (PYTHAGORAS THEOREM)



e.g.4

L.I.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \right\}$$

e.g.5

L.D.

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

CONVENTION: $\{\vec{0}_V\}$ IS L.D. IN ANY VECTOR SPACE.

CASES: (1) $S = \{\vec{v}\}$ IN ANY VECTOR SPACE

(a) $\vec{v} \neq \vec{0} \Rightarrow$ L.I.

(b) $\vec{v} = \vec{0} \Rightarrow$ L.D.

(2) $S = \{\vec{v}_1, \vec{v}_2\}$

(a) IF S IS L.D.

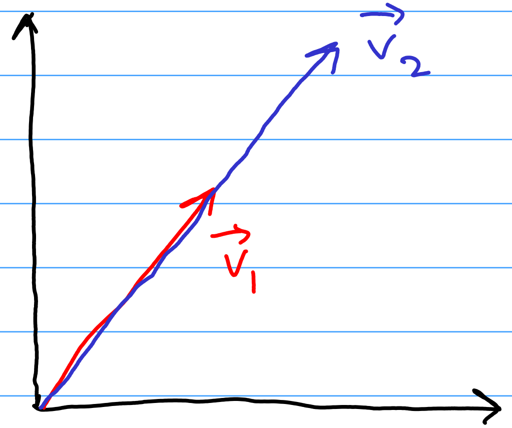
THEN, $\boxed{\vec{v}_1 = k\vec{v}_2 \quad \text{OR} \quad \vec{v}_2 = c\vec{v}_1 \quad (k, c \in \mathbb{R})}$ ⊛

N.B. : TWO EQUATIONS UNLESS $\vec{v}_1$ OR $\vec{v}_2 = \vec{0}$.

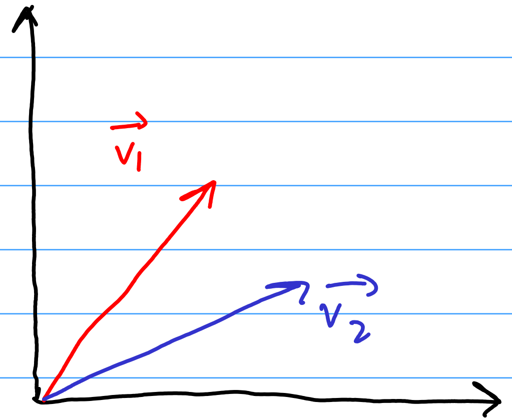
IN FACT $\vec{0}$ IS ALWAYS A LINEAR COMBINAT.

$$\vec{0} = 0\vec{v}_1 + 0\vec{v}_2 + \ldots + 0\vec{v}_k$$

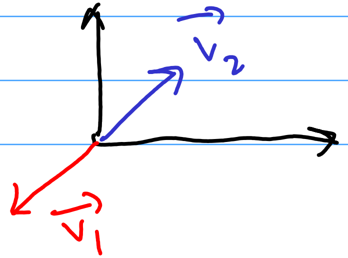
(b) IF ⊛ DOESN'T HOLD THEN S IS L.I



L. D.

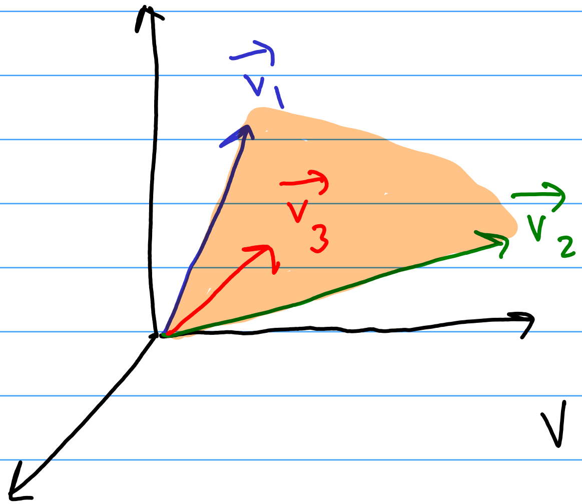


L. I.



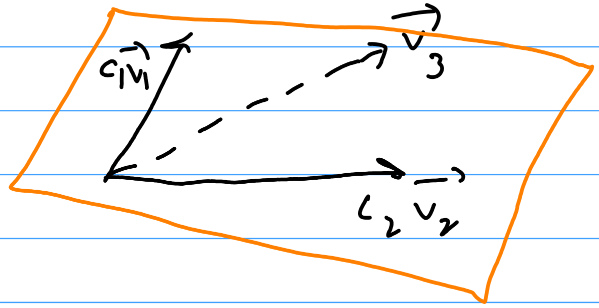
LIE ON SOME PLANE

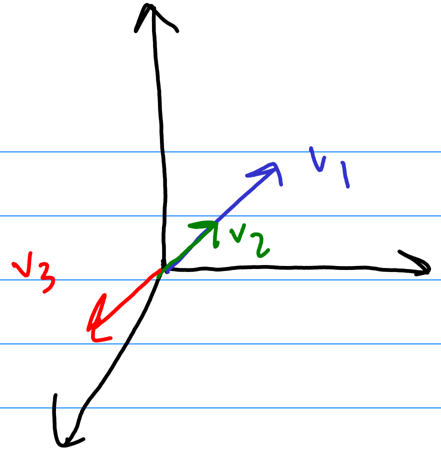
(3) $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IS L.D. $\iff$ $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IS $\boxed{\text{COPLANAR}}$



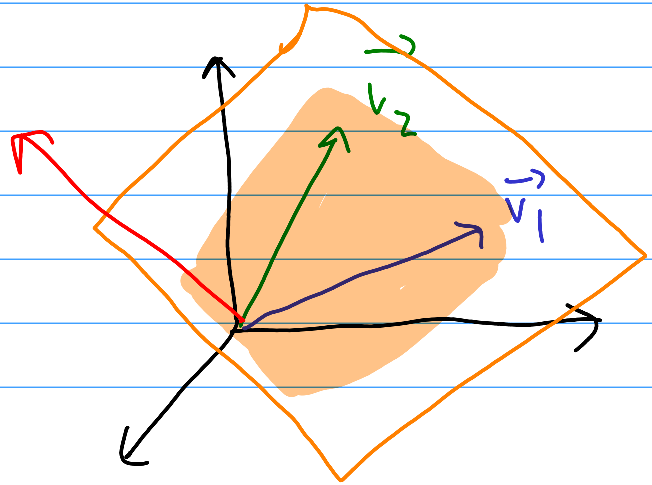
$V = \mathbb{R}^3$

$\vec{v}_3 = c_1\vec{v}_1 + c_2\vec{v}_2$





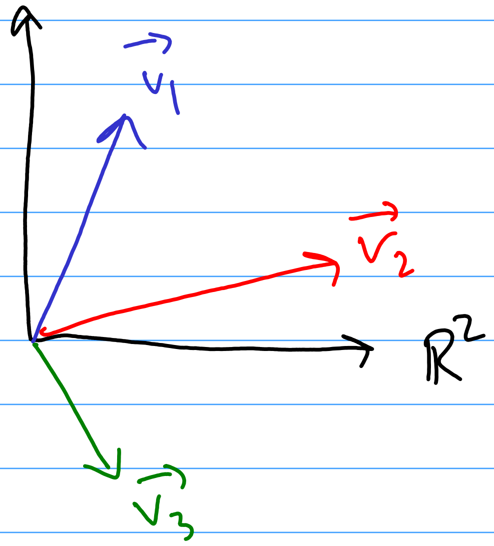
ON THE SAME LINE



L. I.

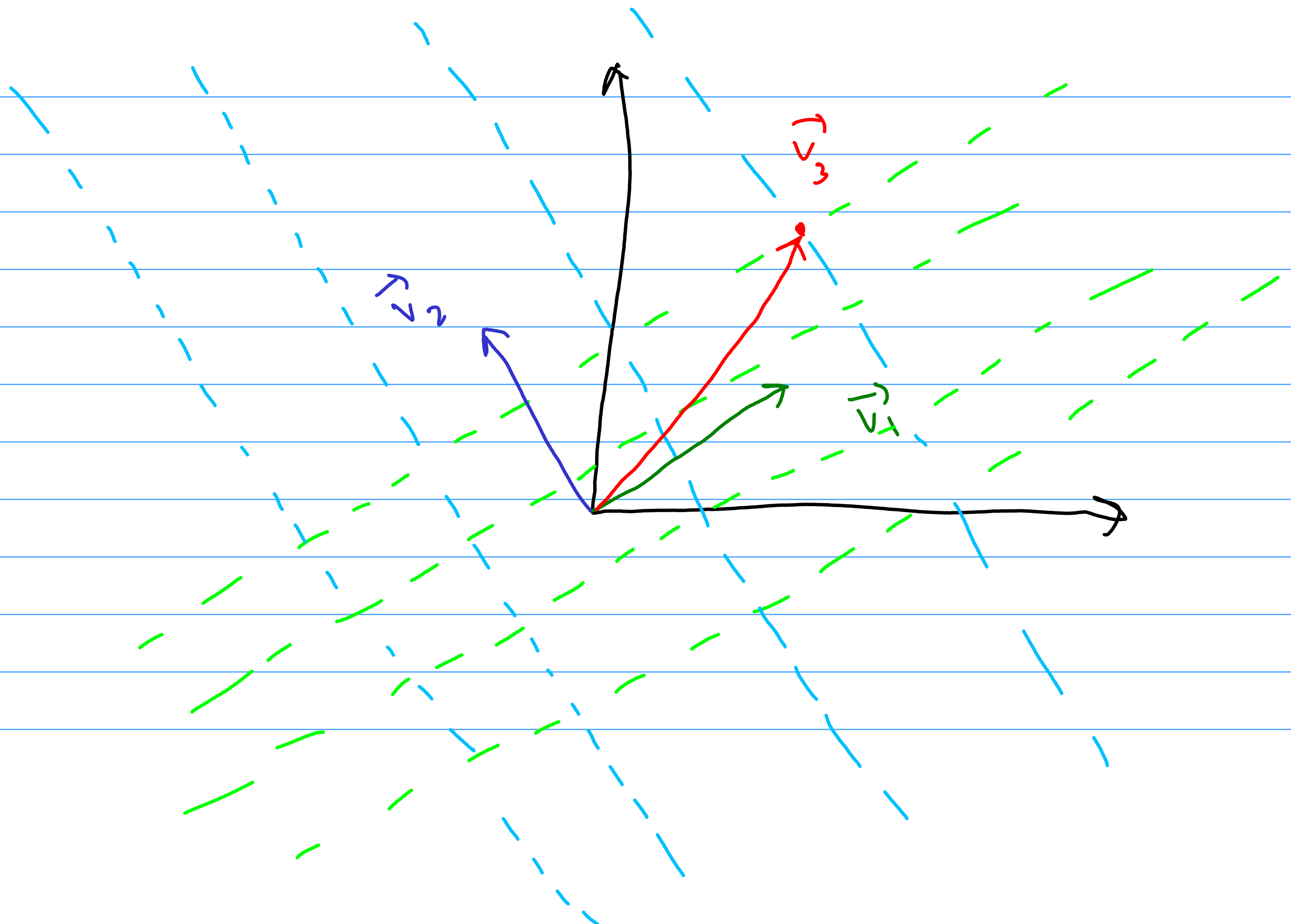
$\mathbb{R}^3$

Q CAN $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IN $\mathbb{R}^2$ BE L.I.?



ANS: NO!

BECAUSE THEY LIE IN THE PLANE $\mathbb{R}^2$.

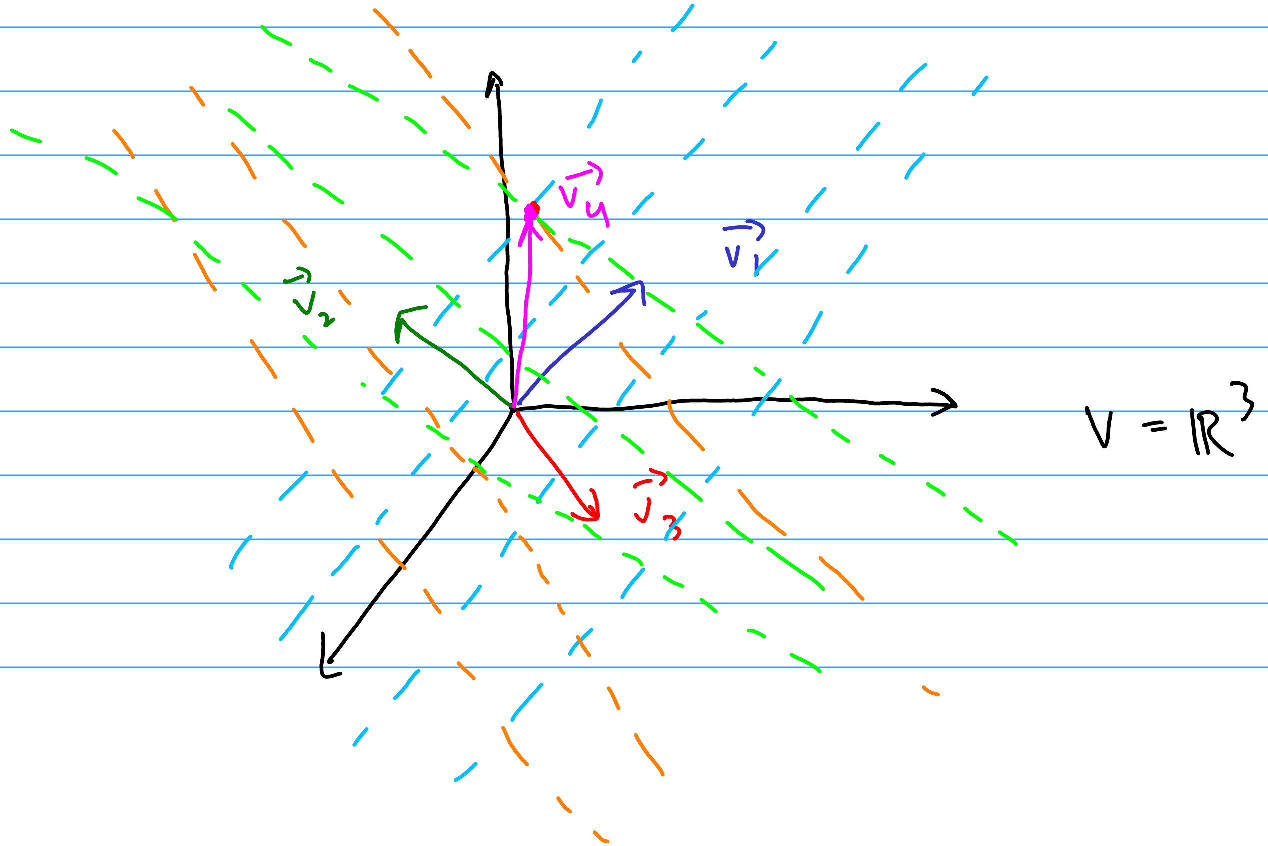


$\vec{v}_3$
$\vec{v}_2$
$\vec{v}_1$

$V = \mathbb{R}^3$

Q: CAN $\{v_1, v_2, v_3, v_4\}$ BE L.I.?

ANS: NO.



OBSERVE: FOR $V = \mathbb{R}^n$ WITH $n = 1, 2, 3.$

THEN $n+1$ VECTORS ARE ALWAYS DEPENDENT.

DIMENSION $\rightarrow$ TO BE EXPLAINED.

DEFN: $(V = \mathbb{R}^n)$ $\quad S = \{\vec{v}_1, \dots, \vec{v}_k\}$

$$\text{Span } S = \{c_1\vec{v}_1 + \dots + c_k\vec{v}_k : c_1, \dots, c_k \in \mathbb{R}\}$$

= SET OF ALL LINEAR COMBINATIONS

$v \in \text{Span } S \iff \vec{v}$ IS L.D. ON S

$$\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k$$

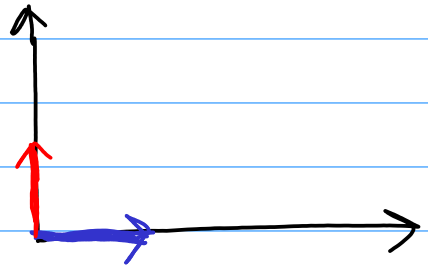
FOR SOME $c_1, \dots, c_k \in S$

e.g. $\quad V = \mathbb{R}^2$

(a) $\quad S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

$$\operatorname{Span} S = \left\{ c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} : c_1, c_2 \in \mathbb{R} \right\}$$

$$= \mathbb{R}^2$$

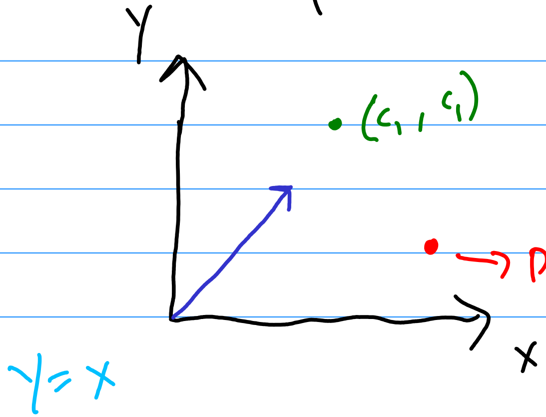


e.g.

$$V = \mathbb{R}^2$$

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$$\operatorname{Span} S = \left\{ c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_1 \end{bmatrix} : c_1 \in \mathbb{R} \right\}$$

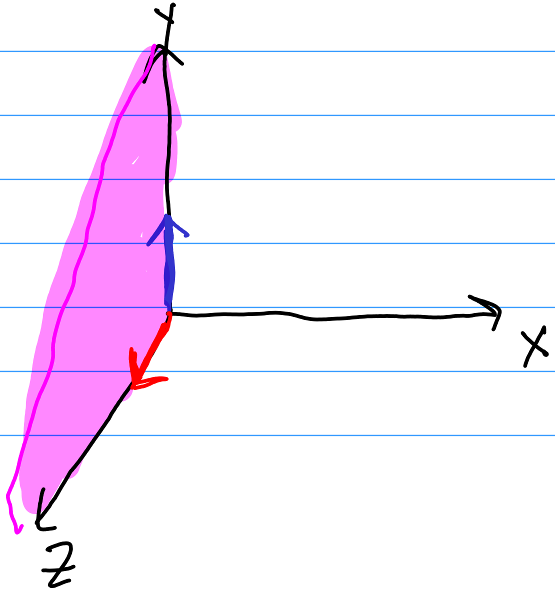


DOES NOT LIE ON Y=X

e.g. $V = \mathbb{R}^3$

$$S = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\operatorname{Span} S = \left\{ c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 0 \\ c_1 \\ c_2 \end{bmatrix} \right\}$$



$V = \mathbb{R}^3$

$$V = M_{2\times 2}(\mathbb{R})$$

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\operatorname{Span} S = \left\{ c_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right.$$

$$= \left. \begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} : c_j \in \mathbb{R} \right\}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$V = P_2(\mathbb{R}) = \{ a_0 + a_1 t + a_2 t^2 : a_j \in \mathbb{R} \}$$

(a) $S = \{ 1, t \}$

$$\operatorname{Span} S = \{ c_1 \cdot 1 + c_2 \cdot t = c_1 + c_2 t : c_j \in \mathbb{R} \}$$

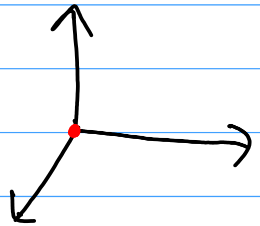
$$= P_1(\mathbb{R})$$

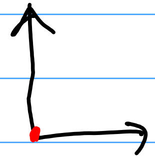
(b) $S = \{ 1 \}$

$$\operatorname{Span} S = \{ c_1 \cdot 1 = c_1 : c_1 \in \mathbb{R} \} = P_0(\mathbb{R})$$

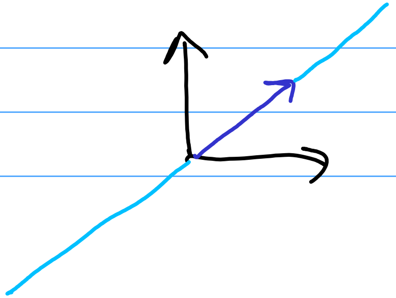
CASES: $V = \mathbb{R}^n$

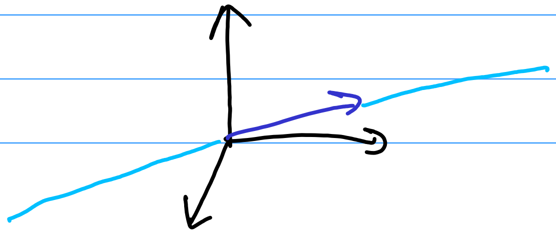
(0) $S = \{\vec{0}\}$ $\quad$ $\operatorname{Span} S = \{\vec{0}\}$





(1) $S = \{\vec{v}\}$ $\quad$ $\vec{v} \neq 0$ $\quad$ $\operatorname{Span} S = \{c_1 \vec{v} : c_1 \in \mathbb{R}\}$

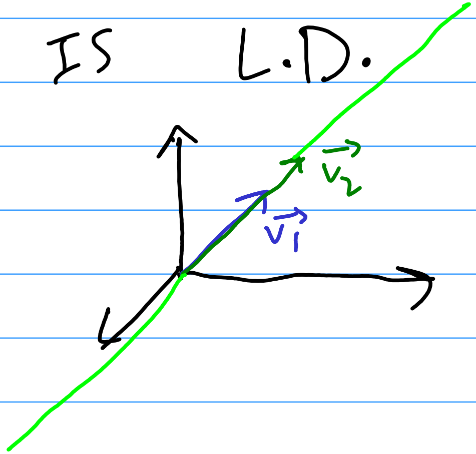




(2) $S = \{\vec{v}_1, \vec{v}_2\}$

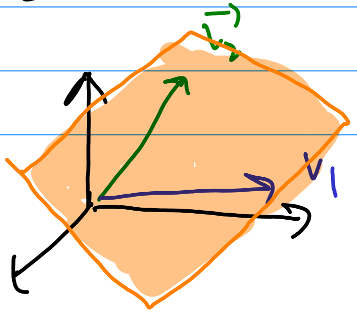
$\text{Span } S = \{c_1\vec{v}_1 + c_2\vec{v}_2\}$

(a) S IS L.D.



(b) S IS L.I.

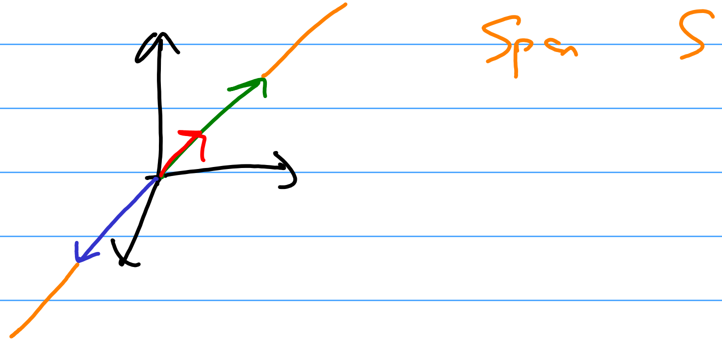
$\text{Span } S = \{c_1 v_1 + c_2 v_2\}$



(3) $S = \{ v_1, v_2, v_3 \}$

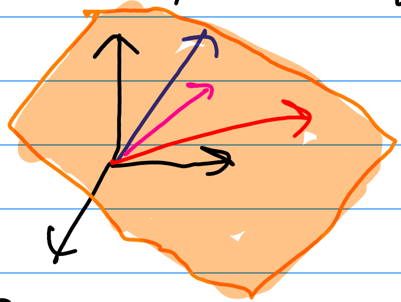
(a) COLLINEAR

L. D.



(b) COPLANAR BUT NOT COLLINEAR

L. D.



(c) NOT COPLANAR

L. I.