MA 35100 (SPRING 2025, SEC: 130)

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SECTION WEB PAGE: https://www.math.purdue.edu/~sahay5/spring2025/ma35100/index.html

NOTE: ALL IMAGES ARE FROM THE TEXTBOOK (R.C. PENNEY, 4th ed.)

ANNOUNCEMENTS / NOTES

1. CLASSES ARE BACK IN-PERSON FROM TUESDAY.

2. NO OFFICE HOURS OR H.W. THIS WEEK

   ¡ PLEASE WORK ON H.W. 2 NOW ANYWAY ! (DUE: WED, FEB 5th)

3. RECORDING OF CLASS WILL BE UPLOADED TO KALTURA.

4. TRY TO KEEP VIDEOS ON.

DEFN: A SET S IS LINEARLY INDEPENDENT (L.I.) IF NONE OF THE ELEMENTS IS A LINEAR COMBINATION OF THE OTHERS.

CONVERSELY, A SET IS LINEARLY DEPENDENT (L.D.) IF ONE OF ITS ELEMENTS IS A LINEAR COMBINATION OF THE OTHERS

e.g. 1 L.D.

$$\left\{ \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$$

$$4 \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

e.g. 2 L.I.

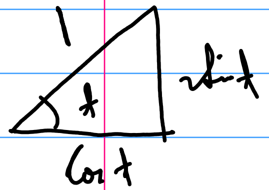
$$\left\{ \begin{bmatrix} \boxed{1} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \boxed{1} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \boxed{1} \end{bmatrix} \right\}$$

e.g.3

L.D.

$$\{1, \sin^2 t, \cos^2 t\} \qquad V = \mathcal{F}(\mathbb{R})$$

$$\sin^2 t + \cos^2 t = 1 \qquad [\text{PYTHAGORAS THEOREM}]$$



$$\sin^2 t = 1 - \cos^2 t$$

e.g.4

L.I.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \right\} //$$

e.g.5

L.D.

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} //$$

CONVENTION: $\{\vec{0}_V\}$ IS L.D. IN ANY VECTOR SPACE.

CASES: (1) $S = \{\vec{v}\}$ (IN ANY VECTOR SPACE)

(a) IF $\vec{v} = \vec{0}$ THEN S IS L.D.

(b) IF $\vec{v} \neq \vec{0}$ THEN S IS L.I.

(2) $S = \{\vec{v}_1, \vec{v}_2\}$

(a) IF S IS L.D.

$\circledast$ — $\boxed{\vec{v}_2 = k\vec{v}_1 \quad \text{OR} \quad \vec{v}_1 = c\vec{v}_2 \quad (k, c \in \mathbb{R})}$

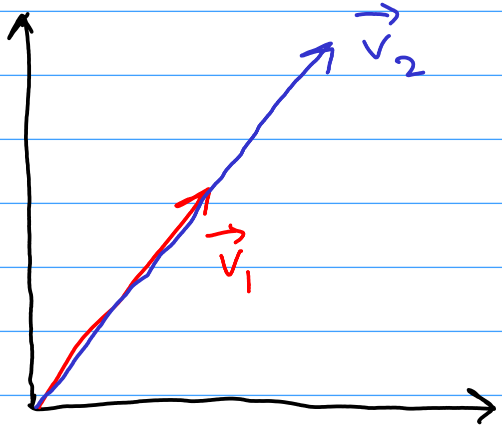
IF EITHER $\vec{v}_1$ OR $\vec{v}_2 = 0$, L.D.

N.B. : $\vec{0}$ IS ALWAYS A LINEAR COMBINATION

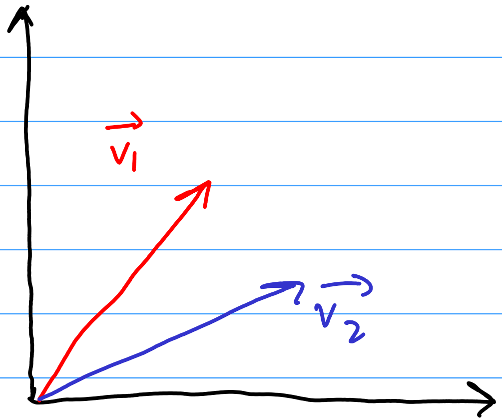
$$\vec{0} = 0\vec{v}_1 + 0\vec{v}_2 + \cdots + 0\vec{v}_k$$

(b) IF $\circledast$ IS FALSE FOR EVERY $k, c \in \mathbb{R}$,
THEN S IS L.I.

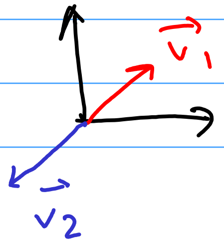
$V = \mathbb{R}^2$

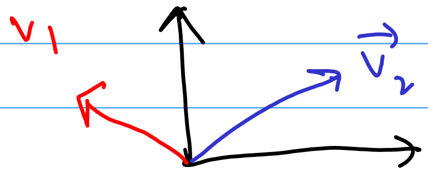


L. D.



L. I.



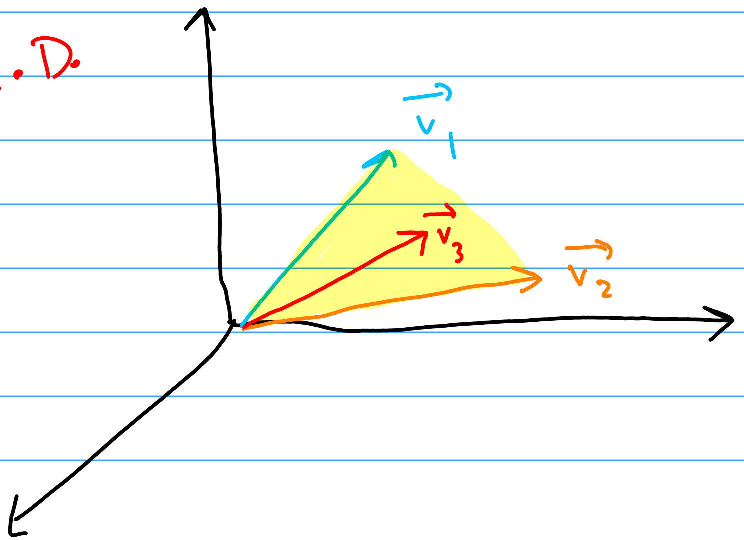


$V = \mathbb{R}^n$

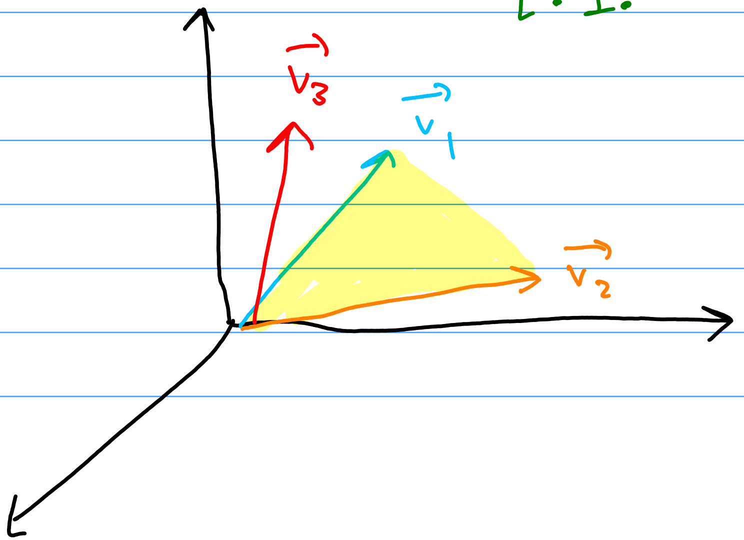
(3) $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IS L.D. $\Longleftrightarrow$ $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IS $\underbrace{\text{COPLANAR}}_{\text{LIE IN THE SAME PLANE}}$

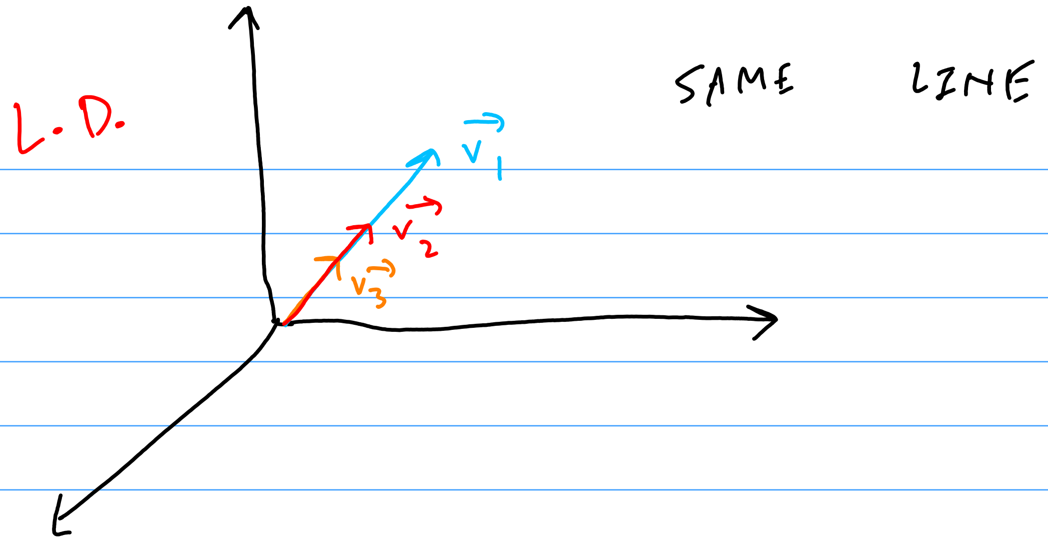
$\vec{v}_3 = c_1\vec{v}_1 + c_2\vec{v}_2$

L.D.



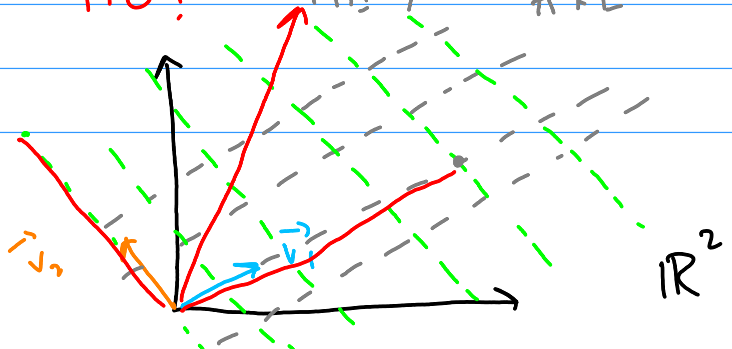
L.I.





Q : CAN $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IN $\mathbb{R}^2$ BE L.I. ?

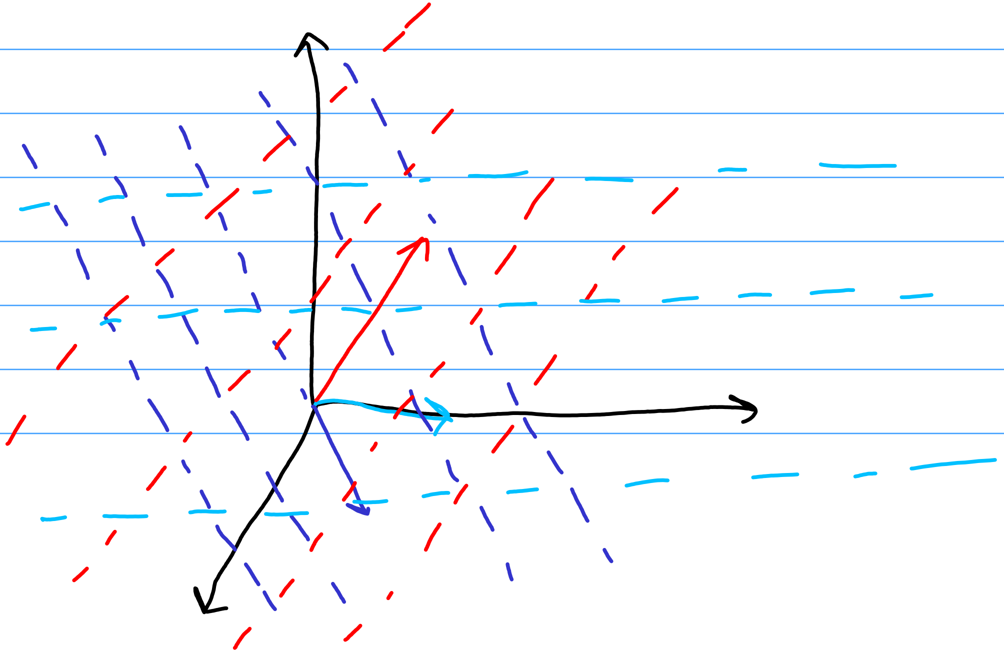
A : NO! THEY ARE ALWAYS COPLANAR!



$V = \mathbb{R}^3$

Q. CAN $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$ BE L. I.?

A. NO!



OBSERVE: FOR $V = \mathbb{R}^n$ WITH $n = 1, 2, 3$
THE NO SET WITH $n+1$ ELEMENTS
CAN BE L.I.

DIMENSION $\longrightarrow$ EXPLAINS THIS OBSERVATION.

DEFN: $(V = \mathbb{R}^n)$ $S = \{ \vec{v}_1, \dots, \vec{v}_k \}$

$$\operatorname{Span} S = \{ c_1 \vec{v}_1 + \dots + c_k \vec{v}_k : c_1, \dots, c_k \in \mathbb{R} \}$$

= SET OF ALL LINEAR COMBINATIONS

$v \in \operatorname{Span} S \iff \vec{v}$ IS L.D. ON S

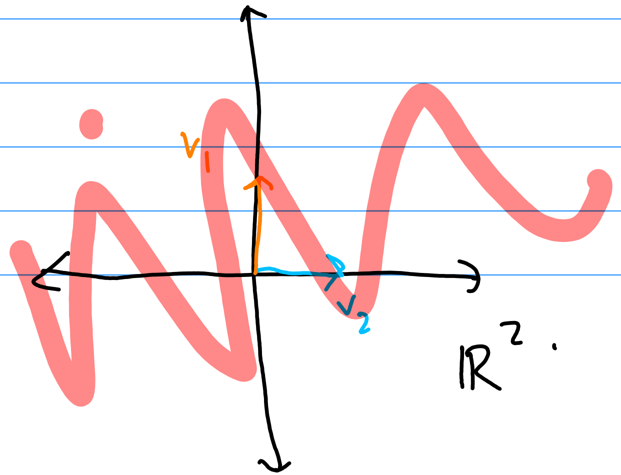
$v = c_1 \vec{v}_1 + \dots + c_k \vec{v}_k$ FOR SOME $c_1, \dots, c_k \in \mathbb{R}$

e.g. $V = \mathbb{R}^2$

$$S = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \overset{v_2}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \right\}$$

$$c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_2 \\ c_1 \end{bmatrix}$$

$\operatorname{Span} S = \mathbb{R}^2$



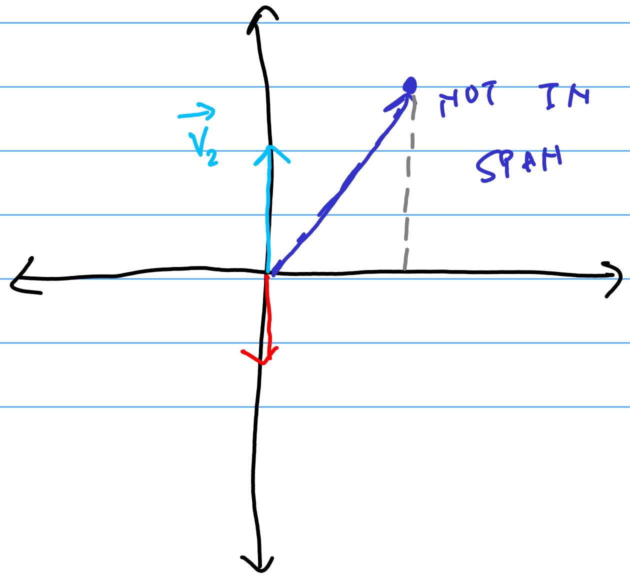
$\mathbb{R}^2$.

$V = \mathbb{R}^2$

$S = \left\{ \vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Span $S \rightarrow$ Y-AXIS.

$c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ c_2 \end{bmatrix}$



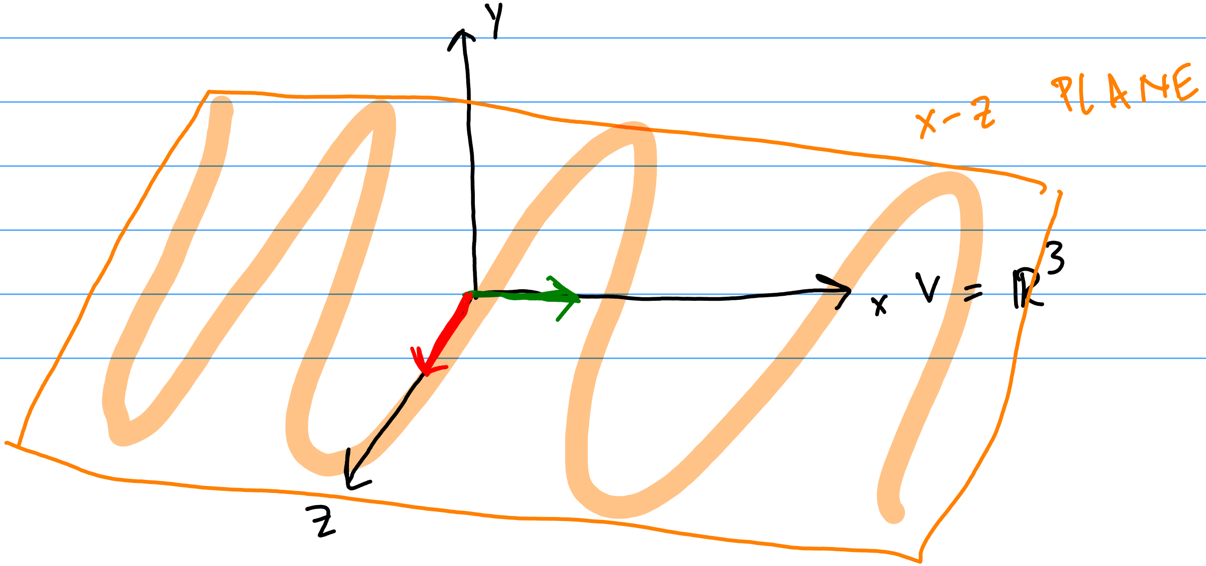
$V = \mathbb{R}^2$

$$V = \mathbb{R}^3$$

$$S = \left\{ \overset{\vec{v}_1}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}, \overset{\vec{v}_2}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \right\}$$

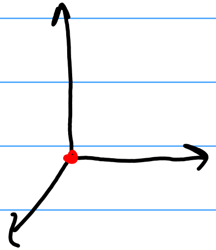
$$\text{Span } S \rightarrow c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

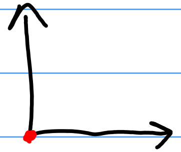
$$= \begin{bmatrix} c_1 \\ \boxed{0} \\ c_2 \end{bmatrix}$$



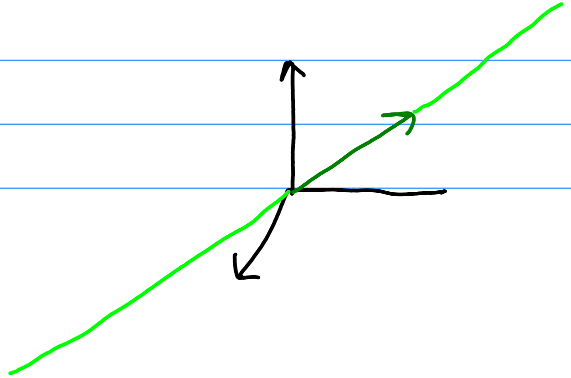
CASES: $V = \mathbb{R}^n$

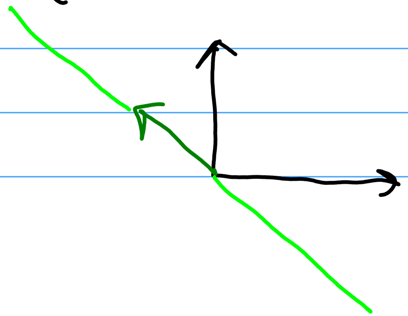
(0) $S = \{\vec{0}\} \Rightarrow \operatorname{Span} S = \{\vec{0}\}$





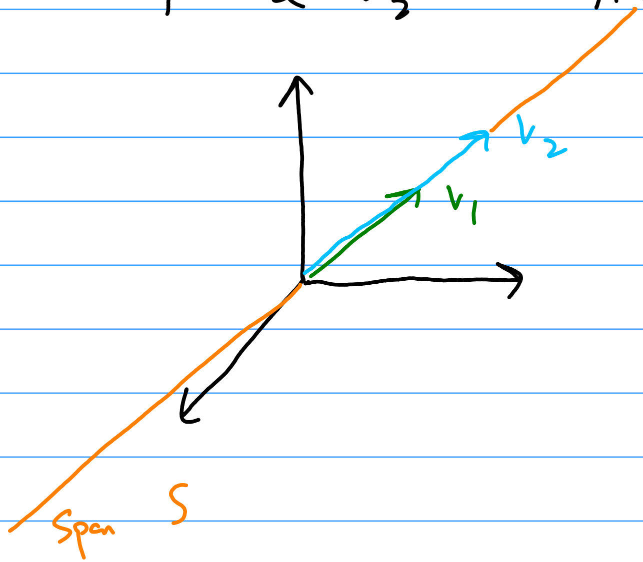
(1) $S = \{\vec{v}\}$ $(\vec{v} \neq \vec{0})$ $\operatorname{Span} S = \{c\vec{v} : c \in \mathbb{R}\}$





(2) $V = \{\vec{v}_1, \vec{v}_2\}$

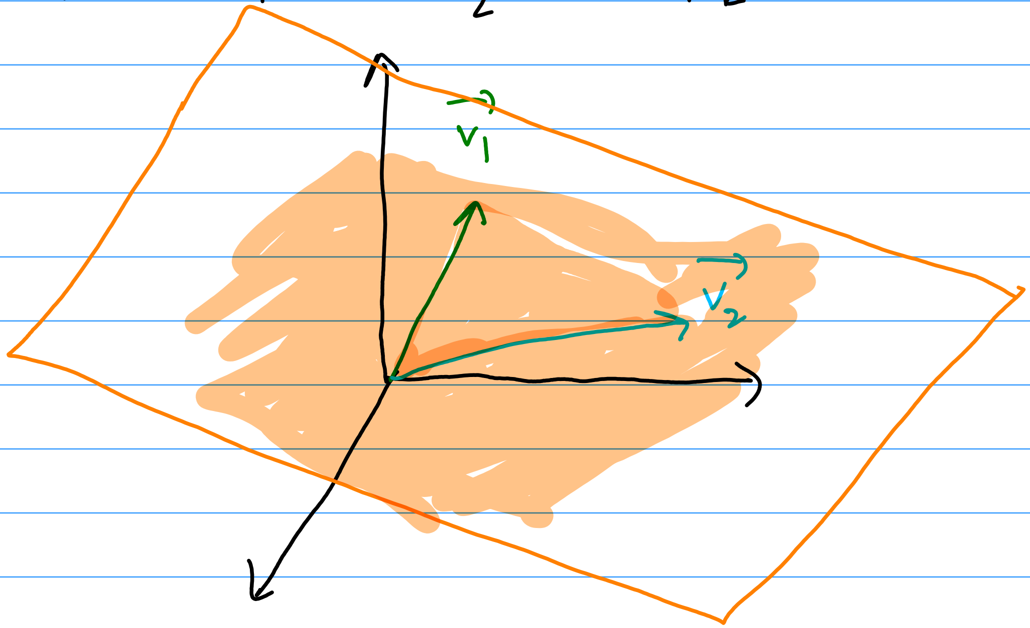
(a) $\vec{v}_1$ & $\vec{v}_2$ ARE L.D.



$\text{Span } S = \{c_1\vec{v}_1 + c_2\vec{v}_2 : c_1, c_2 \in \mathbb{R}\}$

(a) $V = \{\vec{v_1}, \vec{v_2}\}$

(b) $\vec{v_1}$ & $\vec{v_2}$ ARE L.I.



$\text{Span } S = \{c_1\vec{v_1} + c_2\vec{v_2} : c_1, c_2 \in \mathbb{R}\}$

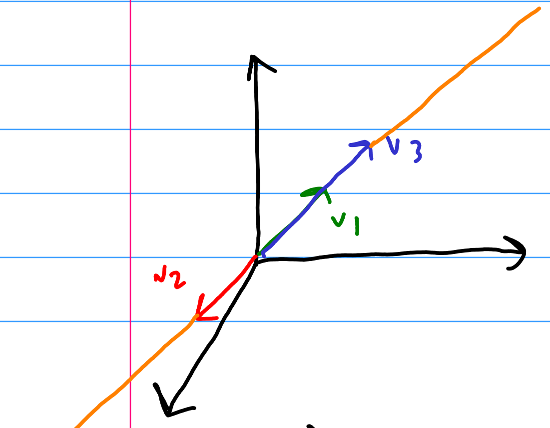
(3) $S = \{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$

$\text{Span } S = \{c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} : c_1, c_2, c_3 \in \mathbb{R}\}$

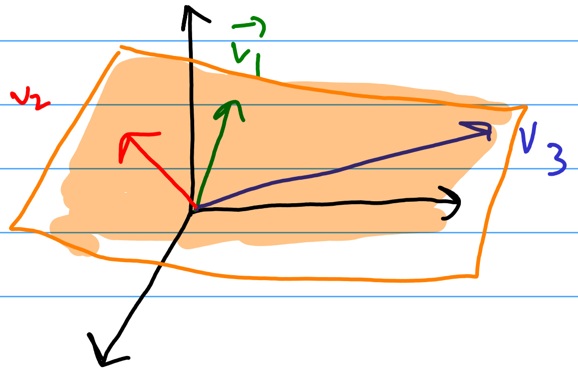
(a) $\vec{v_1}, \vec{v_2}, \vec{v_3}$ ARE L.D. AND IN A LINE

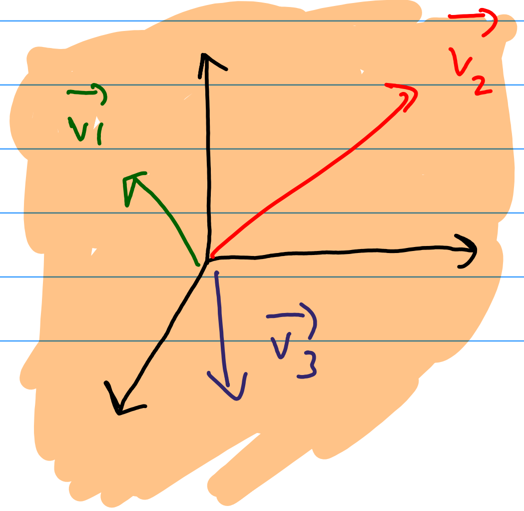
(b) $\vec{v_1}, \vec{v_2}, \vec{v_3}$ ARE L.D. BUT NOT IN A LINE

(c) $\vec{v_1}, \vec{v_2}, \vec{v_3}$ ARE L.I.



(a)





$$V = P_2(\mathbb{R}) = \{ a_0 + a_1 t + a_2 t^2 : a_0, a_1, a_2 \in \mathbb{R} \}$$

(a) $S = \{1, t\}$

$$\operatorname{Span} S = \{ c_1 \cdot 1 + c_2 \cdot t \}$$
$$= \{ c_1 + c_2 t \} = P_1(\mathbb{R})$$

(b) $S = \{1\}$

$$\operatorname{Span} S = \{ c_1 \} = P_0(\mathbb{R})$$

$$V = M_{2\times 2}(\mathbb{R})$$

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\text{Span } S = \left\{ c_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$