

# MA 35100: Elementary Linear Algebra

## Midterm 1

February 27th, 2025

- This is the exam for §§130-131 of MA 351.
- The exam will be 60 minutes long.
- There are 15 pages.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use theorems proved in class provided you accurately state them before using them.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- When the time is over, put down your writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: \_\_\_\_\_

PUID: \_\_\_\_\_

### Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: \_\_\_\_\_

QUESTION	VALUE	SCORE
1	15	
2	5	
3	5	
4	5	
5	5	
6	5	
TOTAL	40	

**1. (15 points)**

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

- (a) The dimension of the space  $P_5(\mathbb{R})$  is 5.
- (b) Any homogeneous system of linear equations with 3 equations and 4 variables has no solutions.
- (c) The subset  $W = \{(t, t^2) : t \in \mathbb{R}\}$  is a subspace of  $\mathbb{R}^2$ .
- (d) If  $V$  is a vector space and  $W \subset V$  is a subspace, then the zero vector  $0_V \in W$ .
- (e) If  $n$ ,  $r$  and  $r^\#$  have their usual meanings for a linear system, then we always have  $r, r^\# \leq n$ .
- (f) If  $A$  and  $B$  are row-equivalent, then they have the same row-space.

- (g) The matrix  $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$  is in row-reduced echelon form.

- (h) The set of matrices  $A$  in  $M_{2 \times 3}$  with the property that  $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \vec{0}$  is a subspace of  $M_{2 \times 3}$ .

- (i) For any vector space  $V$  and any  $\vec{v} \in V$ ,  $0\vec{v} = \vec{0}$ .

- (j) The set of vectors  $\{(1, 1, 1), (2, 3, 1), (0, 1, -1)\}$  is linearly independent.

**2. (5 points)**

For the following two statements determine whether they are true or false. In either case, provide appropriate justification for your answer (e.g., a counterexample or a brief proof).

- (a) The space of symmetric  $3 \times 3$  matrices  $\{A \in M_{3 \times 3}(\mathbb{R}) : A^T = A\}$  has dimension 6.
- (b) An  $m \times n$  matrix  $A$  is nonsingular if and only if its nullity is 0.

(Continued)

**3. (5 points)** For what values of  $k$  does the following system have no solution, a unique solution, and infinitely many solutions respectively?

$$x + y + kz = 3$$

$$2x + 6z = 5$$

$$x - y + k^2z = k$$

Final Answer:

(Continued)

4. (5 points) Find all solutions to the following system of equations.

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$3x_1 - x_2 + 2x_3 - x_4 = 7$$

$$-x_1 - 2x_2 - x_3 - 2x_4 = 2$$

$$x_2 + x_4 = -2$$

Final Answer:



(Continued)

**5. (5 points)**

Find a basis for  $\text{Span } S$ , where

$$S = \left\{ \begin{bmatrix} 22 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 3 \\ 12 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -27 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \right\}.$$

Final Answer:

(Continued)

**6. (5 points)** Compute the rank and nullity of the following matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 5 & 7 & 11 \\ -3 & -3 & -3 & -3 & -3 \\ 22 & 23 & 25 & 27 & 31 \\ -5 & 10 & 0 & -10 & 5 \end{bmatrix}$$

Final Answer:

(Continued)

Scratch Work

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