MA 35100: Elementary Linear Algebra

Midterm 2

April 9th, 2025

- The exam will be 60 minutes long.
- There are 14 pages.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use any theorem proved in class provided you accurately state it before using it.
- When the time is over, all students must put down their writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ______________________________

PUID: ______________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	15	
2	5	
3	5	
4	5	
5	5	
6	5	
TOTAL	40	

1. (15 points)

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers (however, please see the next question).

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) If A and B are both $n \times n$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

(b) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is injective then it must be surjective.

(c) If $\mathcal{B} = \{\vec{b}_1, \cdots, \vec{b}_n\}$ is an ordered basis for $\mathbb{R}^n$, then the coordinate matrix $P_{\mathcal{B}}$ has the basis element $\vec{b}_j$ as its jth column.

(d) If $\det A = 2$, $\det B = 2$ and $\det C = 1$, then $\det(A^T B^{-1} C^{100}) = 1$.

(e) If A, B, C are 2×2 invertible matrices with $AB = AC$, then $B = C$.

(f) If $A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix}$ for $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^2$, then the area of the parallelogram generated by $\vec{v}_1$ and $\vec{v}_2$ is $\det A$.

(g) If A is an $n \times n$ matrix such that the system of equations $A\vec{x} = \vec{0}$ has only the trivial solution $\vec{x} = \vec{0}$, then $\det A = 0$.

(h) The differentiation map $D : P_n(\mathbb{R}) \to P_n(\mathbb{R})$ given by $D(p) = p'$ has trivial kernel.

(i) If A and B are row-equivalent, then $\det A = \det B$.

(j) The inverse of an upper-triangular matrix is always lower-triangular.

2. (5 points)

For the following two statements determine whether they are true or false. In either case, provide appropriate justification for your answer (e.g., a counterexample or a brief proof).

(a) If A is a 4×3 matrix and B is a 3×4 matrix, then $\det AB = 0$.

(b) If $A^{-1} = A^T$ for a invertible matrix A, then $\det A = 1$.

(Continued)

3. (5 points) Suppose that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -10.$$

Compute the following determinant.

$$\begin{vmatrix} a-c+d-f & d-f & g-i \\ 7c+7f & 7f & 7i \\ b/2+e/2 & e/2 & h/2 \end{vmatrix}$$

Final Answer:

(Continued)

4. (5 points) Compute the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 4 & 9 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Answer:

(Continued)

5. (5 points)

Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ be an ordered basis of $\mathbb{R}^2$ with

$$\vec{b}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \qquad \vec{b}_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}.$$

Compute the point matrix $P_{\mathcal{B}}$ and the coordinate matrix $C_{\mathcal{B}}$. Further, for

$$\vec{x} = \begin{bmatrix} 11 \\ 0 \end{bmatrix}, \qquad [\vec{y}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -2 \end{bmatrix},$$

compute $[\vec{x}]_{\mathcal{B}}$ and $\vec{y}$.

Final Answer:

(Continued)

6. (5 points) Consider the linear map $T : P_3(\mathbb{R}) \to \mathbb{R}^3$ given by

$$T(p(t)) = (p(0), p'(1), p''(2)).$$

Given the ordered bases $\mathcal{B}_1 = \{1, t, t^2, t^3\}$ of $P_3(\mathbb{R})$ and $\mathcal{B}_2 = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ of $\mathbb{R}^3$, find the matrix of the transformation T in these bases. Is this map an isomorphism?

Final Answer:

(Continued)

Scratch Work