Question Bank for Final Exam

MA 425/52500 (Spring 2026, §001)

April 27, 2026 (Updated: April 28)

Notes

- There will be four types of questions on the final exam: T/F (True or False), Proofs, MCQs (Multiple Choice Questions), and Long Answer Questions (LAQs).
- The structure of the final exam will **(tentatively!)** be as follows:
  - 10 T/Fs (1 points each = 10 points total).
  - 2 Proofs (4-6 points each = 10 points total)
  - 4 MCQs (5 points each = 20 points total). Each MCQ will have 5 options, of which only one is correct.
  - 3-4 LAQs (5-10 points each depending on complexity = 30 points total)
- The final exam will be worth 70 points total.
- This is a sample question bank for LAQs and proofs.
- LAQs similar to the ones that showed up in Midterm 1 and Midterm 2 may also show up in the final; for these also, please see the archives.

Long Answer

Problem L1. Compute the number of zeroes of $z^3+4z^2-(1+i)z+1$ inside the disc $|z|<1$.

Problem L2. Compute the integral

$$\int_0^\infty \frac{x}{x^3+1}dx$$

by means of complex methods.

Problem L3. Find the number of zeroes of z^3-z+1 inside the right half-plane $\mathrm{Re}(z)>0$.

Problem L4. Compute the integral

$$\int_{-\infty}^{\infty} \frac{x^k \cos x}{x^4 + x^\ell + 1} dx.$$

for $k \in \{0, 1, 2, 3\}$ and $\ell \in \{-2, 0, 2\}$.

[**Note:** this problem has been worded the way it is to give you as much practice as possible. In an exam, only one choice of the pair (k, ℓ) will be expected of you.]

Problem L5. Let $a > b > 0$. Compute the integral

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$$

by relating it to a contour integral.

Problem L6. Compute the integral

$$\int_0^{\infty} \frac{\log t}{(t^2+1)^k} dt$$

for $k \in \{1, 2\}$ by complex methods.

[**Note:** this problem has been worded the way it is to give you as much practice as possible. In an exam, only one choice of k will be expected of you.]

Problem L7. Consider the function

$$f(z) = \frac{z}{(z^2+1)\sin z}.$$

Find and classify all singularities of f in the disc $|z| < 2\pi$. For each such singularity w, compute the residue $\mathrm{Res}(f; w)$ and hence compute the integral

$$\int_{|z|=2\pi} f(z)dz.$$

[**Note:** here we follow the usual convention that when a circle is written as in this integral, it is traversed counter-clockwise once.]

Problem L8. Find all maximal annuli $U_j = \{z \in \mathbb{C} : r_j < |z| < R_j\}$ centered at the origin such that the function

$$g(z) = \frac{z}{(1-z^3)}.$$

has a Laurent series expansion on U_j. Compute these expansions.

[**Note:** For practice, here are some alternative choices of g. For practice for a different type of problem, I suggest trying to compute the residues of these functions at their various poles.]

(a)

$$\frac{1+z}{(1-z)(2+z)}$$

(b)

$$\frac{3z}{(1+4z)(4+z)}$$

(c)

$$\frac{1}{1+z+z^2}$$

(d)

$$\frac{z}{(1+z^2)(1+z+z^2)}$$

(e)

$$\frac{1}{(z+1)(z+2)(z+3)}$$

Proof Questions

Problem P1. Let $f(z)$ be an entire function satisfy $|f(z)| \leqslant |z|^2$ for all $z \in \mathbb{C}$. Show that there is a constant $a \in \mathbb{C}$ with $|a| \leqslant 1$ such that $f(z) = az^2$ for every $z \in \mathbb{C}$.

Problem P2. Let f be an entire function which has a pole of order m at ∞. Show that f must be a polynomial of degree m.

Problem P3. Let f be analytic on a bounded domain D and continuous on its closed $\overline{D}$. Further, suppose that $f(z) \neq 0$ for $z \in \overline{D}$ and further that $|f(z)| = M$ for $z \in \partial D$ where M is a fixed positive real number. Show that f is a constant function. You may use without proof the fact that if f is analytic and $|f|$ is constant then f is constant.

Problem P4. Prove that all the roots of the polynomial

$$p(z) = z^7 - 5z^3 + 12 = 0$$

lie in the annulus $U = \{z \in C : 1 < |z| < 2\}$.