MA 42500/52500: Elements Of Complex Analysis

Midterm 1

February 16th, 2026

- This is the exam for §001 of MA 425/525.
- The exam will be 60 minutes long.
- There are 14 pages.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use theorems proved in class provided you accurately state them before using them.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- When the time is over, put down your writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ______________________________

PUID: ______________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	10	
2	10	
3	6	
4	5	
5	5	
6	4	
TOTAL	40	

1. (10 points)

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) The function $f(z) = z^2$ is injective on $\mathbb{C} \setminus \{0\}$.

(b) A singleton set $\{w\}$ with one element $w \in \mathbb{C}$ is always open.

(c) We have that $\operatorname{Arg}(z) + \operatorname{Arg}(-\overline{z}) = \pi$ where Arg is the principal branch of the logarithm.

(d) For $z, w \in \mathbb{C}$,

$$|z + w|^2 = |z|^2 + |w|^2 + 2\operatorname{Re}(z\overline{w}).$$

(e) The set $\{x + iy \in \mathbb{C} : -1 < x, y < 1\}$ contains none of its boundary points.

(f) Suppose that U is a convex open set in the complex plane such that $-1, 1 \in U$. Then the line segment joining -1 to 1 is in U, and hence, in particular, $0 \in U$.

(g) If $z + i\overline{z} = 0$ then $z = 0$.

(h) The range of the function $g(z) = e^z$ is $\mathbb{C} \setminus \{0\}$.

(i) If $\{z_j\}$ and $\{w_j\}$ are sequences such that

$$\lim_{j\to\infty} z_j = \lim_{j\to\infty} w_j = L,$$

then $z_j w_j \longrightarrow L^2$ as $j \longrightarrow \infty$.

(j) The function $h : \hat{\mathbb{C}} \longrightarrow \mathbb{C}$ given by

$$h(z) = \begin{cases} \frac{|z|+1}{|z|+2} & \text{if } z \neq \infty \\ 0, & \text{if } z = \infty, \end{cases}$$

is continuous at all points in $\hat{\mathbb{C}}$.

2. (10 points)

For the following two statements determine whether they are true or false. In either case, provide appropriate justification for your answer (e.g., a counterexample or a proof as necessary).

(a) The set

$$\{z \in \mathbb{C} : 1 < \operatorname{Im}(z) < 2\}$$

is open.

(b) Suppose that the infinite series

$$\sum_{j=1}^{\infty} z_j$$

converges. Then

$$\lim_{n\to\infty} \frac{z_{n+1}}{z_n} = 0.$$

(Continued)

3. (6 points) Compute the following limits or determine that they do not exist:

(a)

$$\lim_{z \to 2\pi i} \frac{e^{3z} - 1}{e^z - 1} = \boxed{}.$$

(b)

$$\lim_{z \to \infty} \frac{z^3 - 1}{3z^3 + iz - 2} = \boxed{}.$$

(c)

$$\lim_{n \to \infty} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^n = \boxed{}.$$

(Continued)

4. (5 points) What is the image of the function $g : D \to \mathbb{C}$ where

$$D = \{z \in \mathbb{C} : 1 < |z| < 2\}$$

and

$$g(z) = z^3.$$

Draw a picture representing the image.

Final Answer:

(Continued)

5. (5 points) Let $h : U \longrightarrow \mathbb{C}$ be a branch of the logarithm defined by the following properties:

- U is obtained by deleting the non-negative imaginary axis $\{iu : u \in \mathbb{R} \text{ and } u \geqslant 0\}$ from $\mathbb{C}$.
- The point -1 is chosen to have $h(-1) = i37\pi$.

Compute $\sqrt[4]{16} = 16^{\frac{1}{4}}$ where $a^z = \exp(z \log a)$ using this branch of the logarithm.

Final Answer:

(Continued)

6. (4 points) For which $w \in \mathbb{C}$ does the following series converge? For the values that it does converge, find a formula for the sum.

$$\sum_{k=1}^{\infty} (2w + i)^k.$$

Final Answer:

(Continued)

Scratch Work

Scratch Work