MA 42500/52500: Elements Of Complex Analysis

Midterm 2

March 30th, 2026

- This is the exam for §001 of MA 425/525.
- The exam will be 60 minutes long.
- There are 18 pages.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use theorems proved in class provided you accurately state them before using them.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- When the time is over, put down your writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ____________________

PUID: ____________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ____________________

QUESTION	VALUE	SCORE
1	10	
2	6	
3	4	
4	6	
5	4	
6	4	
7	6	
TOTAL	40	

1. (10 points) Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers. You may assume that if an integral is being taken over a curve, then the curve in question is piece-wise smooth.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) The function $u(x, y) = x^3 - 3xy^2$ is harmonic.

(b) For every $R \in [0, \infty]$ and $z_0 \in \mathbb{C}$, there is a power series which has radius of convergence R and center z_0.

(c) If γ_1 and γ_2 are closed curves in $\mathbb{C}$ starting and ending at the same point, then $\gamma_1 \oplus \gamma_2$ is also a closed curve in $\mathbb{C}$.

(d) If $\gamma : [a, b] \longrightarrow D$ is a smooth curve and $u : D \longrightarrow \mathbb{C}$ is a function, then the line integral of u over γ is defined by

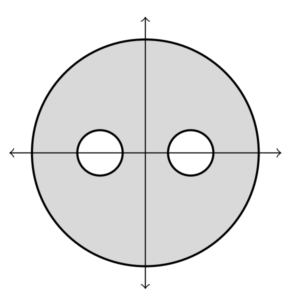
$$\int_\gamma u(z)\, dz = \int_a^b u(\gamma(t))\gamma'(t)\, dt.$$

(e) The radius of convergence of the power series

$$\sum_{n=0}^{\infty} 3^n n^2 (z-3)^n,$$

is 1/3.

(f) The shaded region in the following diagram is simply connected.



(g) If $f : U \longrightarrow \mathbb{C}$ and $g : U \longrightarrow \mathbb{C}$ are holomorphic functions on the domain U, then the quotient function $h : U \longrightarrow \mathbb{C}$ defined by $h(z) = f(z)/g(z)$ is always holomorphic by the quotient rule.

(h) If $\tilde{\gamma}$ is the reversal of γ, then

$$\int_\gamma f(z)\,dz = \int_{\tilde{\gamma}} f(z)\,dz.$$

(i) For any closed curve γ which does not pass through the origin and any $m \in \mathbb{N}$,

$$\int_\gamma z^m\,dz = 0.$$

(j) Let $D = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disk. Then every holomorphic function on D has an anti-derivative.

2. (6 points)

For the following two statements determine whether they are true or false. In either case, provide appropriate justification for your answer (e.g., a counterexample or a proof as necessary).

(a) Let U be the domain $\{z \in \mathbb{C} : 1 < |z| < 3\}$ and let γ be the circle $|z| = 2$ traversed clockwise once. Let $f : U \to \mathbb{C}$ be an analytic function. Then, by Cauchy's theorem,

$$\int_\gamma f(z)\,dz = 0.$$

(b) If $f : \mathbb{C} \longrightarrow \mathbb{C}$ is a holomorphic function such that for some $r \in \mathbb{R}$, $\operatorname{Re} f(z) = r$ for every z (i.e., the real part of f is constant). Then $f(z)$ must be constant.

3. (4 points) Compute the harmonic conjugate of $u(x, y) = e^x \cos y - y$ which satisfies $v(0, 0) = 2$. Identify the holomorphic function given by $f(z) = u(x, y) + iv(x, y)$ for $z = x + iy$.

Final Answer:

(Continued)

4. (6 points)

Compute the following integrals using Cauchy's theorem or Cauchy's integral formula:

(a)

$$\int_{|z|=1} \frac{e^z + 1}{z}\, dz = \boxed{}.$$

(b)

$$\frac{1}{2\pi i} \int_{|z+3|=1} \frac{8}{(z^2 - 1)(z + 3)}\, dz = \boxed{}.$$

(c)

$$\int_{|z-2|=1} \frac{e^z \sin^2 z}{z^2 + iz}\, dz = \boxed{}.$$

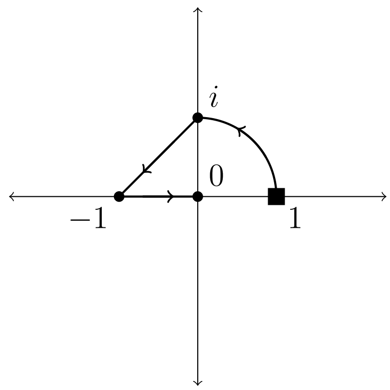
(Continued)

5. (4 points) Compute the power series of $g(z) = (1+z)^{-2}$ around $z = 0$. What is the radius of convergence of this power series?

Final Answer:

(Continued)

6. (4 points) Find a parametrization for the following curve:



Final Answer:

(Continued)

7. (6 points) Compute the following integral by using Cauchy's theorem or Cauchy's integral formula:

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}.$$

Final Answer:

(Continued)

Scratch Work

Scratch Work