MA 42500/52500: Elements Of Complex Analysis

Sample Midterm 2

March 26th, 2026

- This is the sample exam for §001 of MA 425/525.
- The exam will be 60 minutes long.
- There are 18 pages.
- Unless stated otherwise, **show all your work and provide justifications.** You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- You may use theorems proved in class provided you accurately state them before using them.
- No calculators, phones, electronic devices, books, notes are allowed during the exam.
- When the time is over, put down your writing instruments immediately.
- Violating these instructions will be considered an act of academic dishonesty. Any act of dishonesty will be reported to the Office of the Dean of Students. Penalties for such behaviour can be severe and may include an automatic F in the course.

NAME: ______________________________

PUID: ______________________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________________

QUESTION	VALUE	SCORE
1	10	
2	6	
3	3	
4	4	
5	7	
6	4	
7	6	
TOTAL	40	

1. (10 points)

Determine if the following statements are true or false. Fill in your answers in the following table. You do not need to justify your answers. You may assume that if an integral is being taken over a curve, then the curve in question is piece-wise smooth.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

(a) The function $u(x, y) = x^2 + y^2$ is harmonic.

(b) For every $R \in [0, \infty]$ and $z_0 \in \mathbb{C}$, there is a power series which has radius of convergence R and center z_0.

(c) If γ_1 and γ_2 are simple curves in $\mathbb{C}$ such that the endpoint of γ_1 is the beginning point of γ_2, then $\gamma_1 \oplus \gamma_2$ is also a simple curve in $\mathbb{C}$.

(d) If $\gamma : [a, b] \longrightarrow D$ is a smooth curve and $u : D \longrightarrow \mathbb{C}$ is a function, then the line integral of u over γ is defined by

$$\int_\gamma u(z)\, dz = \int_a^b u(\gamma(t))\gamma''(t)\, dt.$$

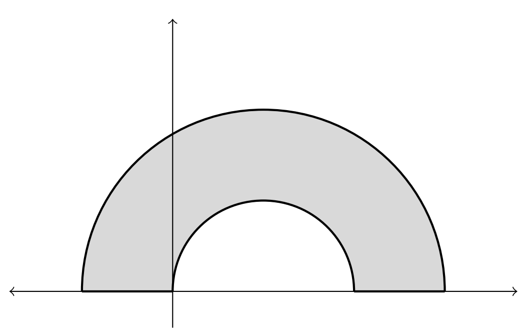
(e) If R is the radius of convergence of the power series

$$\sum_{n=0}^{\infty} a_n (z - z_0)^n,$$

then

$$\frac{1}{R} = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

(f) The shaded region in the following diagram is simply connected.



(g) If $f : D \longrightarrow \mathbb{C}$ and $g : U \longrightarrow D$ are holomorphic functions for domains U, D, then $f \circ g : U \longrightarrow \mathbb{C}$ is also holomorphic by the chain rule.

(h) Let $f : D \to \mathbb{C}$ be a nonvanishing function defined on a simply connected domain. Then, there exists a holomorphic function $h : D \to \mathbb{C}$ which is a branch of $\log f(z)$ in the sense that $f(z) = \exp\Big(h(z)\Big)$ for every $z \in D$.

(i) If $\tilde{\gamma}$ is the reversal of γ, then

$$\left|\int_\gamma f(z)\, dz\right| = \left|\int_{\tilde{\gamma}} f(z)\, dz\right|.$$

(j) For any closed curve γ which avoids the origin and any $m \in \mathbb{Z}$,

$$\int_\gamma z^m\, dz = 0.$$

2. (6 points)

For the following two statements determine whether they are true or false. In either case, provide appropriate justification for your answer (e.g., a counterexample or a proof as necessary).

(a) Let U be the domain $\{z \in \mathbb{C} : 1 < |z| < 3\}$ and let γ be the circle $|z| = 2$ traversed clockwise once. Let $f : U \to \mathbb{C}$ be an analytic function. Then, by Cauchy's theorem,

$$\int_\gamma f(z)\, dz = 0.$$

(b) If $f : \mathbb{C} \longrightarrow \mathbb{C}$ is a holomorphic function such that for some $\theta \in [0, \pi/2)$, $\arg f(z) = \theta$ for every z (i.e., the argument of f is constant and lies in $[0, \pi/2)$). Then $f(z)$ must be constant.

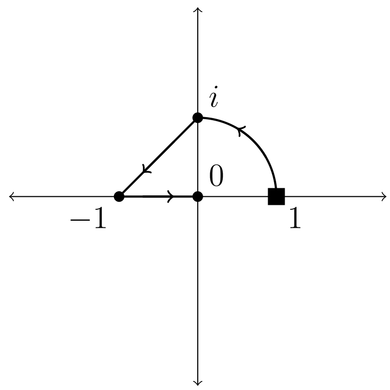
(Continued)

3. (3 points) Compute the harmonic conjugate $u(x,y) = x^2 - y^2 + x$ which satisfies $v(0,0) = 1$.

Final Answer:

(Continued)

4. (4 points) Find a parametrization for the following curve:



Final Answer:

(Continued)

5. (7 points) Compute the following integral by using Cauchy's theorem or Cauchy's integral formula:

$$\int_{-\infty}^{\infty} \frac{\cos 5x}{4+x^2}\,dx.$$

Final Answer:

(Continued)

6. (4 points) Compute the Taylor series of $g(z) = z^{-1}$ around $z = 1$. What is the radius of convergence of this Taylor series?

Final Answer:

(Continued)

7. (6 points)

Compute the following integrals using Cauchy's theorem or Cauchy's integral formula:

(a)

$$\int_{|z|=1} \frac{e^z + 1}{z}\, dz = \boxed{}.$$

(b)

$$\frac{1}{2\pi i} \int_{|z|=2} \frac{8}{(z^2 - 1)(z + 3)}\, dz = \boxed{}.$$

(c)

$$\int_{|z-2|=1} \frac{\sin z}{z^2 + iz}\, dz = \boxed{}.$$

(Continued)

Scratch Work

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