MA161 Quiz 10 Solutions

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Problem 10.1. Find the exact values of the following expressions:

(a) $\sin(\pi/6)$,

(b) $\cos(\pi/6)$,

(c) $\arctan(1/\sqrt{3})$,

(d) $\arcsin(1/2)$.

Solution. For part (a) and (b) $\sin(\pi/6) = 1/2$ and $\cos(\pi/6) = \sqrt{3}/2$ (these are just values you should know—see the Wikipedia image of the **Unit Circle** for example). For parts (c) and (d), you can use what you know to determine the values of the expressions. For example, in the case of (c),

$$\frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \frac{\sin(\pi/6)}{\cos(\pi/6)} = \tan(\pi/6)$$

so $\arctan(1/\sqrt{3}) = \arctan(\tan(\pi/6)) = \pi/6$ by definition. Similarly for (d), $\sin(\pi/6) = 1/2$ so $\arcsin(1/2) = \arcsin(\sin(\pi/6)) = \pi/6$. ☺

Problem 10.2. Find an equation of the tangent line to the curve

$$y = \sec(x) \text{ at the point } (\pi/3, 2).$$

Solution. Remember that if we know the slope m and a point (x_0, y_0) on the tangent line, then the tangent line is

$$y - y_0 = m(x - x_0);$$

do not forget this. We already have our x_0 and y_0, they are $\pi/3$ and 2, respectively. All we need to do now is find m, which we do below

$$y' = (\sec x)' = \sec x \tan x$$

so

$$y'(\pi/6) = \sec(\pi/3)\tan(\pi/3) = 2\sqrt{3}.$$

Therefore, the tangent line is

$$y - 2 = 2\sqrt{3}(x - \pi/3). \qquad ☺$$

Problem 10.3. Simplify the expression

$$\tan(\arcsin(x)).$$

Solution. This problem was verbatim an exercise from Lesson 14. The Pythagorean Identity is very important and you should remember it; in case you do not, here it is

$$\sin^2(x) + \cos^2(x) = 1.$$

Now, remember that $\tan(x) = \sin(x)/\cos(x)$ and also, from the Pythagorean Identity, $\cos(x) = \sqrt{1 - \sin^2(x)}$ so

$$\tan(\arcsin(x)) = \frac{\sin(\arcsin(x))}{\cos(\arcsin(x))} = \frac{\sin(\arcsin(x))}{\sqrt{1 + \sin^2(\arcsin(x))}}.$$

But since sin and arcsin are inverses of each other,

$$\frac{\sin(\arcsin(x))}{\sqrt{1 - \sin^2(\arcsin(x))}} = \frac{x}{\sqrt{1 - x^2}}. \qquad ☺$$

Problem 10.4. Find the limit

$$\lim_{x\to\infty} x\sin(\pi/x).$$

Solution. This may have been the trickiest problem in this quiz. Here is the method we were meant to follow. First, notice that

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

so if we make the substitution $u = 1/x$ to our original limit, we now have the equivalent limit

$$\lim_{u\to 0} \frac{\sin(\pi u)}{u}. \tag{$\star$}$$

But we only know that

$$\lim_{t\to 0}\frac{\sin(at)}{at} = 1$$

so we have to multiply Equation ($\star$) by π/π to get it in the form above like so

$$\lim_{u\to 0}\frac{\pi}{\pi}\cdot\frac{\sin(\pi u)}{u} = \pi\left(\lim_{u\to 0}\frac{\sin(\pi u)}{\pi u}\right) = \pi.$$ ☺