MA161 Quiz 11 Solutions

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Problem 11.1. The position for the movement of a particle is given by

$$s(t) = \cos(2t - 2) - \sin(3t - 3),$$

where the position s is measured in feet and t in second.

(a) Find the velocity of the particle after one second; i.e., at time $t = 1$.

(b) Find the acceleration of the particle after one second.

Solution. Remember that the velocity is the first derivative of the position and acceleration the second. That is,

$$\begin{aligned} v(t) &= s'(t) = -2\sin(2t-2) - 3\cos(3t-3), \\ a(t) &= s''(t) = -4\cos(2t-2) + 9\sin(3t-3). \end{aligned}$$

Then finding the quantities I asked for in parts (a) and (b) is a simple matter of plugging in $t = 1$ into the equations above.

For (a), $v(1) = -3$ and for (b), $a(1) = -4$. ☺

Problem 11.2. Suppose $a > 0$ and the tangent line to $y = a^{x^2}$ at $x = 1$ has slope $m = a$. What is a?

Hint: Remember that a tangent line looks like $y - y_0 = m(x - x_0)$.

Solution. You could have done this problem a number of ways. This is the way I intended you to do it. Write

$$y = a^{x^2} = e^{x^2 \ln(a)}.$$

Then, by the Chain Rule,

$$y' = e^{x^2 \ln(a)}(2x \ln(a)).$$

Now, I told you that $y'(1) = a$ so

$$a = y'(1) = e^{1^2 \ln(a)}(2 \cdot 1 \ln(a)) = 2a \ln(a).$$

Therefore $a = e^{1/2}$. ☺

Problem 11.3. Find the derivative of $y = x^{\tan^{-1}(x)}$.

Hint: The derivative of $\tan^{-1}(x)$ is

$$(\tan^{-1}(x))' = \frac{1}{(x^2+1)}.$$

Solution. For this problem, we can use a similar method to the one we employed above. Write

$$y = x^{\tan^{-1}(x)} = e^{\tan^{-1}(x)\ln(x)}.$$

Then explicitly do the Chain Rule. That is, write $g(u) = e^u$ and $h(v) = \tan^{-1}(v)\ln(v)$ for your functions in the composition. Then $y = g(h(x))$ and by the Product Rule,

$$h'(v) = \frac{\tan^{-1}(v)}{v} + \frac{\ln(v)}{v^2+1}$$

and from class $g'(u) = e^u$ so putting these two together with the Chain Rule,

$$y = g(h(x))' = g'(h(x))h'(x) = \left(\frac{\tan^{-1}(x)}{x} + \frac{\ln(x)}{x^2+1}\right)e^{\tan^{-1}(x)\ln(x)}.$$ ☺