MA161 Quiz 13 Solutions

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Problem 13.1. Let $f(x) = x^2 + 2^{x^2}$. Find $f'(-1)$.

Solution. Using previously learned techniques from class, write

$$f(x) = x^2 + e^{x^2 \ln(2)}.$$

Then

$$f'(x) = 2x + 2x\ln(2)2^{x^2}.$$

Thus

$$f'(-1) = -2 - 2\ln(2) \cdot 2 = -2 - 4\ln(2).$$

☺

Problem 13.2. Find an equation of the tangent line to the curve

$$\ln(xy) = 2x^2 - y - 1$$

at the point $(1, 1)$.

Solution. Using indeterminate derivatives, write

$$\begin{aligned}
\ln(xy) &= 2x^2 - y - 1 \\
\frac{xy' + y}{xy} &= 4x - y' \\
\frac{y'}{y} + \frac{1}{x} &= 4x - y' \\
\frac{y'}{y} + y' &= 4x - \frac{1}{x}
\end{aligned}$$

$$\begin{aligned}(\tfrac{1}{y}+1)y' &= 4x - \frac{1}{x}\\ y' &= \frac{4x-\frac{1}{x}}{1+\frac{1}{y}}.\end{aligned}$$

Now, plug in $x = 1$, $y = 1$ into the equation to obtain the slope of the tangent line, which is

$$m = \frac{4-1}{1+1} = \frac{3}{2}.$$

Then, the tangent line itself is

$$y - 1 = \tfrac{3}{2}(x-1).$$ ☺

Problem 13.3. Given that

$$\tfrac{d}{dx}\tan^{-1}(x) = \frac{1}{x^2+1},$$

find the derivative of

$$\tan^{-1}\left(\frac{2}{x^2}\right).$$

Solution. By the Chain Rule, with $f(u) = \tan^{-1}(u)$ and $g(v) = 2x^{-2}$, we have

$$(f(g(x)))' = f'(g(x))g'(x) = \frac{1}{(\frac{2}{x^2})^2+1}\cdot\frac{-4}{x^3}.$$ ☺