MA161 Quiz 14 Solutions

TA: Carlos Salinas

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Problem 14.1. Find the derivative of

(a) $f(x) = x^{5x}$;

(b) $g(x) = x^{2\sin(x)}$.

(c) $h(x) = \ln(\cosh(2x))$.

Hint: Recall that

$$\cosh(x) = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}.$$

Solution. For part (a), we have

$$f(x) = x^{5x} = e^{5x\ln(x)}$$

so

$$f'(x) = 5(1 + \ln(x))x^{5x}.$$

For part (b), we have

$$g(x) = x^{2\sin(x)} = e^{2\ln(x)\sin(x)}$$

so

$$g'(x) = 2\left(\frac{\sin(x)}{x} + \cos(x)\ln(x)\right)x^{2\sin(x)}.$$

For part (c), we have

$$h(x) = \ln(\cosh(2x))$$

gives

$$e^{h(x)} = \cosh(2x)$$

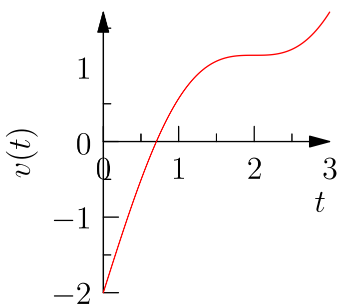
so

$$h'(x)e^{h(x)} = 2\sinh(2x)$$

and $e^{h(x)} = \cosh(2(x))$ so

$$h'(x) = \frac{2\sinh(2x)}{\cosh(2x)} = 2\tanh(2x). \qquad ☺$$

Problem 14.2. The velocity of of a particle $v(t)$ is sketched below as a function of time t



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Determine on which intervals the particle is speeding up and when it is slowing down.

Solution. The function is traveling backward from about $t = 0$ to $t \approx 0.75$, after that, the velocity is positive, so it is traveling forward. Throughout this, it is easy to see from the sketch that the acceleration is positive, but dips to zero at $t = 2$. Therefore, the particle is speeding up from $(0.75, 2) \cup (2, 3)$ and it is slowing down from $(0, 0.75)$. The rest is unknown to us. ☺