MA161 Quiz 15 Solutions

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Problem 15.1. (a) A particle moves along the curve below $y = \sqrt{15 + x^3}$. As it reaches the point $(1, 4)$, the y-coordinate is increasing at a rate of 5 cm/s. How fast is the x-coordinate of the point changing at that instant?

(b) A particle moves along the curve $y = \sqrt{x}$. As the particle passes through $(9, 3)$, its x-coordinate increases at a rate of 3 cm/s. How fast is the distance from the particle to the origin changing at this instant? *Hint*: We want the change in the distance from the origin, the distance at any time is $D(t) = \sqrt{x(t)^2 + y(t)^2} = \sqrt{x(t)^2 + x(t)}$.

Solution. For part (a), as I mentioned in class x and y are dependent on t (although we do not need to know how exactly they depends on it) so

$$y(t) = \sqrt{15 + x(t)^3}$$

and therefore

$$y'(t) = \frac{3x(t)^2 x'(t)}{2\sqrt{15 + x(t)^3}} = \frac{3x(t)^2 x'(t)}{y}.$$

We are given $y'(t)$ at this instant t_0, this is $y'(t_0) = 5$, and at that very same t_0, $x(t_0) = 1$, $y(t_0) = 4$

$$5 = \frac{3 \cdot x'(t_0)}{2 \cdot 4}$$

so

$$x'(t_0) = \frac{2 \cdot 4 \cdot 5}{3} = \frac{40}{3}.$$

For part (b), I gave you the distance explicitly, it is

$$D(t) = \sqrt{x(t)^2 + y(t)^2} = \sqrt{x(t)^2 + x(t)}$$

so

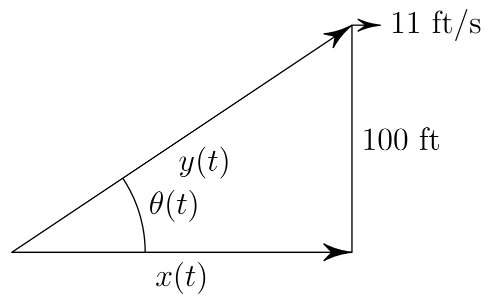
$$D'(t) = \frac{2x(t)+1}{2D(t)} x'(t).$$

Since we know $x'(t_0) = 3$ at some time t_0, we have

$$D'(t) = \frac{2 \cdot 9 + 1}{\sqrt{9^2+9}} 3 = \frac{19 \cdot 3}{\sqrt{90}} = \frac{19}{\sqrt{10}}.$$ ☺

Problem 15.2. A kite 100 ft above the ground moves horizontally at a speed of 11 ft/s. At what rate is the angle (in radians) between the string and the horizontal decreasing when 200 ft of string have been let out?

Hint: The following picture might be useful to you:



Here y is the length of the yarn and the height above the ground remains 100 ft.

Solution. The way to do this problem is to observe that

$$\cot(\theta(t)) = \frac{x(t)}{100}.$$

We know what $x'(t_0)$ is at that instant t_0, and 100 is held constant. Therefore,

$$-\csc^2(\theta(t))\theta'(t) = \frac{x'(t)}{100}$$

so

$$\theta'(t_0) = -\frac{x'(t_0)\sin^2(\theta(t_0))}{100}.$$

Plugging in the values we know,

$$\theta'(t_0) = -\frac{11}{100}\left(\frac{100}{200}\right)^2 = -\frac{11}{400}. \qquad ☺$$