MA161 Quiz 16 Solutions

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Problem 16.1. Use linear approximation to estimate $\sqrt{100.2}$.

Hint: Use the tangent line to $y = \sqrt{x}$ at $x = 100$.

Solution. Using the hint, we find the tangent line to the equation $y = \sqrt{x}$. To do this, we first find its derivative, which is

$$y' = \frac{1}{2\sqrt{x}}.$$

Now we plug in $x_0 = 100$ into the derivative to get

$$m = \frac{1}{2\sqrt{100}} = \frac{1}{20}.$$

All that is left to do is find y_0. This is given by $y_0 = \sqrt{x_0} = \sqrt{100} = 10$. Therefore, the tangent line is

$$y = \frac{1}{20}(x - 100) + 10.$$

Therefore, the approximation is

$$\sqrt{100.2} \approx \frac{1}{20}(100.2 - 100) + 10 = \frac{0.2}{20} + 10 = 10.01.$$ ☺

Problem 16.2. If a snowball melts so that its surface area decreases at a rate of 3 cm^2/min, find the rate at which the radius decreases when the radius is 4 cm.

Hint: The formula for the surface area of a sphere is $S = 4\pi r^2$.

Solution. Using the hint,
$$S = 4\pi r^2$$
so the derivative of S with respect to time is
$$\frac{dS}{dt} = 4\pi(2r)\frac{dr}{dt}.$$
Therefore,
$$\frac{dr}{dt} = \frac{1}{8\pi r}\frac{dS}{dt} = \frac{3}{32\pi}.$$
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Problem 16.3. A ladder 5 ft long is resting against a vertical wall. The top of the ladder is sliding down the wall. At a certain time, y is 4 ft and the angle θ is decreasing at a rate of 2 rad/min. How fast is y decreasing at that time?

Hint: Consider the following picture

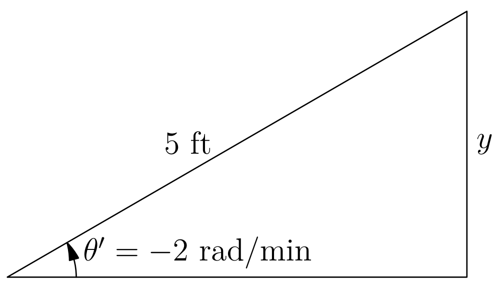

Figure 1: The variable y depends on t so do **not** replace it by 4 when you are setting up the equation which relates θ to y.

Solution. There is no straight forward algorithm for doing **related rates problems**; you just have to make the right choices. In this case, we know the hypotenuse is constant; since this is the length of the ladder and it does not change. Everything else is variable, i.e. subject to change. Therefore, the best way to relate the quantities given is

$$\sin(\theta) = \frac{y}{5}.$$

Then

$$\cos(\theta)\frac{d\theta}{dt} = \frac{1}{5}\frac{dy}{dt}.$$

Therefore,

$$\frac{d\theta}{dt} = \frac{1}{5}\frac{dy}{dt}\frac{1}{\cos(\theta)}.$$

Now we have to plug in what we know into the equation above. We are looking for the rate of y change when $y = 4$. Label the hypotenuse length by h and the adjacent one by x. When $y = 4$, the $x = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$. Therefore,

$$\cos(\theta) = \frac{x}{h} = \frac{3}{5}$$

and $d\theta/dt = -2$. Therefore,

$$\frac{d\theta}{dt} = \frac{1}{5}(-2)\frac{1}{3/5} = -2 \cdot 3 = -6.$$

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