MA161 Quiz 18 Solutions

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Problem 18.1. Find all numbers c in the interval $[0, 25]$ that satisfy the conclusions of Rolle's Theorem for the function $f(x) = \sqrt{x} - x/5$.

Solution. Remember that **Rolle's Theorem** says that if f is differentiable on (a, b) and $f(a) = f(b)$, then $f'(c) = 0$ for some $a < c < b$. The problem at hand actually satisfies the conditions of Rolle's Theorem since $f(x) = \sqrt{x} - x/5$ is differentiable on $[0, 25]$ and $f(0) = f(25) = 0$. Therefore, there must be a c between 0 and 5 such that $f'(0) = 0$. Let's find it:

$$\begin{aligned} f'(x) &= \frac{1}{2\sqrt{x}} - \frac{1}{5}, \\ 0 &= \frac{1}{2\sqrt{x}} - \frac{1}{5} \\ 5 &= 2\sqrt{x} \\ \frac{5}{2} &= \sqrt{x} \\ \frac{25}{4} &= x. \end{aligned}$$

This is the only place where the derivative is zero. ☺

Problem 18.2. Suppose that $2 \leq f'(x) \leq 4$ for all x. What is the maximum possible value of $f(4) - f(2)$?

Solution. By the **Mean Value Theorem**, we have

$$f'(c) = \frac{f(4) - f(2)}{4 - 2}$$

for some $2 < c < 4$. But we know that $2 \leq f'(x) \leq 4$ so that

$$2 \leq \frac{f(4) - f(2)}{4 - 2} \leq 4.$$

Therefore

$$f(4) - f(2) \leq 8.$$

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Problem 18.3. If f is continuous on $[1, 6]$ and differentiable on $(1, 6)$ with $f(6) = 13$ and $f'(x) \geq 2$ for $1 < x < 6$, what is the largest possible value for $f(1)$?

Solution. There was a typo in the original problem. In the original problem, I had "$f(x) \geq 2$" when I should have written "$f(x) \leq 2$."

$$2 \leq \frac{f(6) - f(1)}{6 - 1}.$$

Therefore,

$$10 \leq f(6) - f(1).$$

So

$$3 = f(6) - 10 \geq f(1).$$

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