MA161 Quiz 19 Solutions

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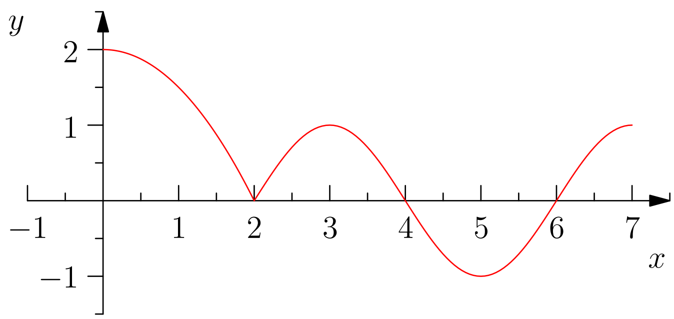
Problem 19.1. Consider the image below



Find the intervals on which

(a) f is increasing;

(b) f is decreasing;

(c) f is concave upward;

(d) f is concave downward.

Solution. For part (a), f is increasing on $(2, 3) \cup (5, 7)$.

For part (b), f is decreasing on $(0, 2) \cup (3, 5)$.

For part (c), f is concave upward on $(4, 6)$.

For part (d), f is concave downward on $(0, 2) \cup (2, 4) \cup (6, 7)$. ☺

Problem 19.2. Suppose that $f'(x) = (x + 2)(x - 5)(x - 6)$. Determine on what intervals f is increasing?

Solution. The best way to solve this problem is to first determine where the zeros of the derivative are; that is why it was given to you factored! The zeros are, by inspection,

$x = -2, 5, 6$ so we have to check the intervals $(-\infty, -2), (-2, 5), (5, 6), (6, \infty)$. Pick a number from each interval, for example, -3, 0, 5.5, 7 and check their values:

$$\begin{aligned} f'(-3) &= (-1)(-8)(-9) < 0, \\ f'(0) &= 2(-5)(-6) > 0, \\ f'(5.5) &= 7.5 \cdot 2.5(-0.5) < 0, \\ f'(7) &= 9 \cdot 2 \cdot 1 > 0. \end{aligned}$$

Therefore, it is increasing on $(-2, 5) \cup (6, \infty)$. ☺

Problem 19.3. Suppose $f''(x) = e^x$ and $f'(1) = 0$. What can you say about $f(1)$?
Hint: Choose **one** of the following options

(a) f has an inflection point at 1

(b) f has a local minimum at 1

(c) f has a local maximum at 1

(d) none of the above.

Solution. By the **Second Derivative Test**, since $f''(1) = e^1 \approx 2.7 > 0$, $f(1)$ must be a local minimum. ☺