MA161 Quiz 20 Solutions

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Problem 20.1. Use **L'Hôpital's Rule** to determine the following limits

(a) $\displaystyle\lim_{x\to 0}\frac{1-\cos(4x)}{x^2}$;

(b) $\displaystyle\lim_{x\to 0^+}(1+\sin(x))^{1/x}$;

(c) $\displaystyle\lim_{x\to 0}\frac{\ln(e^3+3x)-3}{x}$;

(d) $\displaystyle\lim_{x\to 0}(1+2x)^{\cot(x)}$.

Solution. For part (a),

$$\begin{aligned}\lim_{x\to 0}\frac{1-\cos(4x)}{x^2} &= \lim_{x\to 0}\frac{2\sin(4x)}{x}\\ &= \lim_{x\to 0} 8\cos(4x)\\ &= \boxed{8}.\end{aligned}$$

For part (b), let L be the limit we are after. Then

$$\begin{aligned}\ln(L) &= \ln\left(\lim_{x\to 0^+}(1+\sin(x))^{1/x}\right)\\ &= \lim_{x\to 0^+}\frac{\ln(1+\sin(x))}{x}\\ &= \lim_{x\to 0^+}\frac{\cos(x)}{1+\sin(x)}\\ &= 1.\end{aligned}$$

Therefore, $L = e^{\ln(L)} = \boxed{e^1}$.

For part (c),

$$\lim_{x \to 0} \frac{\ln(e^3 + 3x) - 3}{x} = \lim_{x \to 0} \frac{3}{e^3 + 3x} = \boxed{\frac{3}{e^3}}.$$

For part (d),

$$\begin{aligned} \ln(L) &= \lim_{x \to 0} \cot(x) \ln(1 + 2x) \\ &= \lim_{x \to 0} \frac{\ln(1 + 2x)}{\tan(x)} \\ &= \lim_{x \to 0} \frac{2}{(1 + 2x) \sec^2(x)} \\ &= 2. \end{aligned}$$

Therefore, $L = e^{\ln(L)} = \boxed{e^2}$. ☺