MA161 Quiz 22

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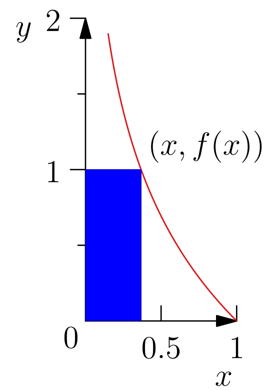
Problem 22.1. A rectangle is formed with one corner at $(0, 0)$ and the opposite corner on the graph of $y = -\ln(x)$, where $0 < x < 1$. What is the largest possible area of such a rectangle? *Hint*: Use the following picture to guide you:



Solution. The length of the sides of the rectangle are x and $f(x)$ so

$$A(x) = xf(x) = -x\ln(x).$$

To optimize this, we need to find the critical points, as we now do

$$A'(x) = -1 - \ln(x).$$

For this to equal zero, we must have $\ln(x) = -1$ which happens when $x = e^{-1}$. Therefore, the largest possible area is

$$A = -e^{-1}\ln(e^{-1}) = e^{-1}.$$

How do we know that this is the maximum? By the Second Derivative Test,

$$A''(x) = -\frac{1}{x}$$

so $A''(e^{-1}) = -e < 0$ which implies that $A(e^{-1})$ is a local maximum. ☺

Problem 22.2. Find the antiderivatives of the following functions:

(a) $f(x) = \frac{1}{2}x^2 - 2x + 5$;

(b) $f(x) = x(3-x)^2$;

(c) $g(t) = \dfrac{7+t+t^2}{\sqrt{t}}$;

(d) $g(t) = 2\sqrt{t} + 8\cos t$

For part (a), we have

$$F(x) = \boxed{\tfrac{1}{6}x^3 - x^2 + 5x + C}.$$

For part (b), $f(x) = x(3-x)^2 = x(x^2 - 6x + 9) = x^3 - 6x^2 + 9x$ so

$$F(x) = \boxed{\tfrac{1}{4}x^4 - 2x^3 + \tfrac{9}{2}x^2 + C}.$$

For part (c), $g(t) = 7t^{-1/2} + t^{1/2} + t^{3/2}$ so

$$G(t) = \boxed{14t^{1/2} + \tfrac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + C}.$$

For part (d),

$$G(t) = \boxed{\tfrac{4}{3}t^{3/2} + 8\sin t}.$$